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**A MODEL FOR DETERMINING ECONOMIC RETENTION LEVELS
OF EXPENDABLE INVENTORY ITEMS**

William R. Harris, et al

**Air Force Institute of Technology
Wright-Patterson Air Force Base, Ohio**

August 1975

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RETENTION LEVELS OF EXPENDABLE
INVENTORY ITEMS

William R. Harris, Captain, USAF
Clare D. Heidler, Captain, USAF

SLSR 34-75B



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This study proposes a model which could be used by the Air Force Logistics Command (AFLC) to compute economic retention levels for expendable type items managed under the D062 Buy Computation System. Cost factors associated with holding and reprocurment actions are identified and evaluated for applicability to the retention situation. Retention models currently used by the Army and the Navy are reviewed with respect to their adaptability for use by AFLC. An economic retention model is developed which balances the discounted holding costs incurred when items are retained and the discounted order costs incurred when items are disposed. The major difference between the proposed model and previous retention level models is in the treatment of storage, obsolescence, and interest costs. Storage cost is applied to the item price when holding costs are computed. Obsolescence and interest costs are applied to the item's net salvage value to represent the opportunity cost incurred when an item is not disposed. A sensitivity analysis shows that the model is relatively sensitive to the storage cost factor and the net salvage factor. The model is relatively insensitive to changes in demand rate and the opportunity cost factor described above.

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A MODEL FOR DETERMINING ECONOMIC RETENTION
LEVELS OF EXPENDABLE INVENTORY ITEMS

A Thesis

Presented to the Faculty of the School of Systems and Logistics
of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the Requirements for the
Degree of Master of Science in Facilities Management

By

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August 1975

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This thesis, written by

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and

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and approved in an oral examination, has been accepted by the undersigned on behalf of the faculty of the School of Systems and Logistics in partial fulfillment of the requirements for the degree of

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CHAPTER I

INTRODUCTION

PROBLEM STATEMENT

Mathematical models provide a framework for management decisions related to the economic quantities of items purchased for Air Force Logistics Command (AFLC) inventories (12:15). However, models such as the Air Force Economic Order Quantity (EOQ) model only indicate the quantity of an item to be acquired. The models do not provide information concerning item quantities to maintain in an inventory in the face of changing demands, technological obsolescence, or deleted requirements (1:41).

Management decisions related to the quantities of items to maintain in an inventory above the required quantities are based on the computation of an economic retention level for each inventory item. Current policy, as outlined in Department of Defense Instruction (DODI) 4100.37, Retention and Transfer of Materiel Assets, specifies that the economic retention level consists of two parts--a quantity equal to the Approved Force Acquisition Objective (AFAO) plus an additional quantity which is more economical to retain than to reprocur at a future date (23:Encl.2,p.1). The AFAO is the quantity of an item

authorized to equip and sustain U. S. Approved Forces and to fulfill commitments to allied forces as designated by the Secretary of Defense.

AFLC's method of computing an economic retention level is stated in AFLC Manual (AFLCM) 57-6, Computation of Requirements for Economic Order Quantity Type Items, as follows:

This level represents . . . zero to five years of stock to be economically retained beyond the Approved Force Acquisition Objective time period. The decision on the number of years of economic retention stocks to hold remains at the AMA [Air Materiel Area] level and should be based on item or weapon system knowledge and not a 'play safe' attitude [19:Par.3-12] .

Computing the economic retention level on the basis of a time period assigned by an item manager rather than on the basis of economic criteria is not in compliance with the provisions of DODI 4100.37.

The purpose of this thesis is to develop a mathematical model, based on economic criteria, which can be used to determine economic retention levels for AFLC inventory items.

JUSTIFICATION

Although compliance with DoD directives concerning inventory management policies is mandatory, the necessity of justifying inventory funding to the DoD, the Office of Manpower and Budget, and the Congress requires that AFLC use a logically developed and economically based method

for computing inventory levels. These two facets of the problem, compliance with DoD directives and justification of inventory funding, have been under study by AFLC for an extended period of time (9). In 1960, Katz (10) produced a model which could be used by AFLC for computing a Serviceable Economic Retention Level for reparable items as a first step in finding problem solutions. However, the subsequent turmoil created in the logistics system by the buildup in the early 1960's in Southeast Asia diverted attention from the problem of determining economic retention levels. With the drawdown in activities in Southeast Asia, DoD has expressed renewed interest in the use of an economic retention formula by the Air Force. The reasons for emphasizing the need for an economic retention level formula are the same as they were in 1960--compliance with DoD directives and justification of funding requests. The requirement now, as then, can be summarized by a quote from a U. S. Navy document published in 1960.

The Navy Supply System has long been faced with the problem of justifying its requests for apportionment of funds for the procurement of more material while its inventory to sales ratios in many categories indicated several years of stock on hand. Much of the long supply picture can be reasonably explained by rollback of material, sudden decreases in certain problems, technological obsolescence, etc. [24:1] .

The development of a retention level formula based on economic criteria can establish compliance with DODI 4100.37 and assist in providing the economic justification needed for inventory funding requests.

SCOPE

Although the concept of economic retention applies to all assets owned by the Air Force, this study is limited to only the expendable type items which are classified by AFLC as EOQ type items. These items are coded with an appropriate Expendability, Recoverability, Reparability, and Cost (ERRC) designation as follows:

1. XB-2. Expendable, nonrecoverable (no repair) items with a projected annual requirement of \$10,000 or more regardless of unit price.
2. XB-3. Expendable, nonrecoverable (no repair) items with a projected annual requirement of \$10,000 or less regardless of unit price.
3. XF-2. Expendable, recoverable (subject to repair but cost and other considerations do not justify automatic return of unserviceable units to the depot) items with a projected annual requirement of \$10,000 or more regardless of unit price.
4. XF-3. Expendable, recoverable (subject to repair but cost and other considerations do not justify automatic return of unserviceable units to the depot) items with a projected annual requirement of \$10,000 or less regardless of unit price.

Expendable type items are managed by AFLC through the use of the D062 EOQ Buy Computation Management Information System (4:5-6).

RESEARCH OBJECTIVE

The objective of this thesis is to develop a mathematical model, based on economic criteria, for

determining an economic retention level for expendable type items managed by AFLC at the depot level.

ORGANIZATION OF THE STUDY

The first step in satisfying the research objective involved the selection of appropriate criteria upon which to develop an economic retention level model. Chapter II describes the derivation of the criteria from a literature review and an analysis of the cost concepts generally applicable to inventory management. In Chapter III, existing economic retention models are examined on the basis of their conformance with the criteria established in Chapter II. Chapter IV details the development of an economic retention level model based on an extension of the computations currently performed in the D062 system. The model's sensitivity to changes in input parameter values will be explored in Chapter V. Chapter VI presents the conclusions drawn from the study, and recommendations based on these conclusions are contained in Chapter VII.

CHAPTER II

CRITERIA SELECTION

ECONOMIC CRITERIA

In the field of inventory management, cost minimization is based on a concept of balancing variable costs (6:30;7:477-483). For example, economic order quantities are computed by balancing the cost of holding an item in inventory and the cost of ordering an item (22:2).

Optimum transfer levels computed by the U. S. Army are based on the concept of balancing the cost of holding an item in inventory and the cost of shipping the item to another activity (18). Economic retention quantities are defined in DODI 4100.37 as the quantities which are more economical to retain for future issues than to replace the future issues by procurement (23:Encl.2,p.2). Therefore, it should be possible to compute economic retention quantities by balancing the cost of holding an item in inventory and the cost to reprocore that item at a future date. Conceptually, an economic retention model appears as shown in Figure 1.

A literature review was conducted in an effort to substantiate the two cost criteria identified in Figure 1. Five studies dealing with economic retention were found

(8;10;17;18;24). All five studies were consistent in their approach to establishing an economic retention level by balancing variable costs of holding an item in inventory and the costs of taking an alternative course of action. The specific model equations will be presented in the next chapter. Here, the focus of attention will be on the economic criteria used.

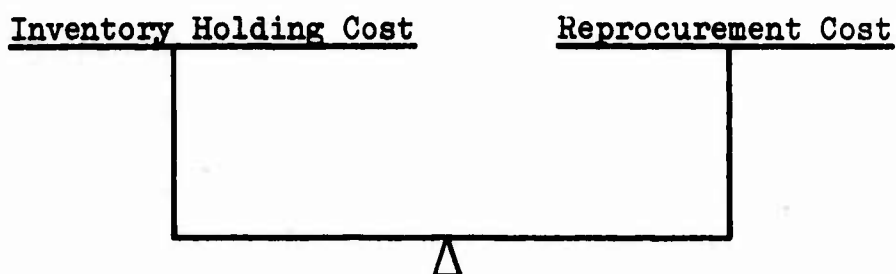


Figure 1

Conceptual Model of Economic Retention

Katz (10:37) developed a model for determining a Serviceable Economic Retention Level for reparable type items. The model he developed was based on a concept of balancing the costs of holding a serviceable unit for a period of years and the cost of repairing an unserviceable unit.

Kaplan (8:i) published a report in 1969 in which he formulated decision rules to be used by depot level managers to decide whether to retain or dispose of stocks in excess of computed stockage requirements. Kaplan's model was designed to balance the value of an item held in inventory for a period of years and the disposal value

of that item at the present time.

A U. S. Army Logistics Management Institute report (18:10), published in 1971, addressed economic retention levels with respect to balancing the cost of holding excess stocks in inventory and the cost of transferring the stocks to other DoD installations.

A U. S. Navy study culminated in the issue of U. S. Navy Bureau of Supplies and Accounts Instruction (BUSANDAINST) 4440.38A, Establishment of Retention Limits and Utilization of Stock Above Protection Limits, which prescribed a mathematical model to be used by the Navy for computing economic retention levels. This model was designed to balance inventory holding costs and item procurement costs (24:1).

Toler (17) published a report in 1972 describing the procedures used at the U. S. Army Mobility Equipment Command to decide on the quantities in excess of the requirements objective which should be returned, retained, or disposed. The procedures he described were based on Kaplan's model modified to facilitate grouping of data by federal stock classes and groups.

The common criterion in these five studies was inventory holding costs. The criterion against which holding costs were balanced varied according to the expressed purpose of each particular formula. Referring to DODI 4100.37, the expressed purpose of economic retention is to determine which is more economical: to

retain or to dispose and reprocore at a later date (23:Encl.2,p.2). In view of the studies previously cited and the defined purpose of economic retention, the concept of computing an economic retention level by balancing holding costs and reprocorement costs appears justified.

The following sections of this chapter will examine the broad criteria established above in greater detail in an effort to establish the factors upon which an economic retention level model should be developed.

Inventory Holding Costs

The known studies concerning economic retention have in common a thorough analysis of inventory holding costs. The discussions of inventory holding costs were primarily derived from an analysis of holding costs related to the determination of economic order quantities. Therefore, a review of the economic order quantity literature was conducted to examine the current values used for inventory holding costs and the concepts upon which these costs were based.

Holding cost rates for Air Force expendable items are currently assessed at a fixed annual rate of 24 percent of an item's purchase price (2:3). The components of the holding cost rate are storage costs, interest costs, and obsolescence costs which include inventory depletion due to losses and deterioration (22:Encl.4,p.1).

Within DoD, storage costs have been assessed at one percent of an item's purchase price. AFLC's one

percent value has been reviewed by the General Accounting Office (GAO) and appears sufficiently justified in DODI 4140.39, Procurement Cycles and Safety Levels of Supply for Secondary Items (22:Encl.4,p.3;25:23). In addition, recent research at the U. S. Air Force Academy evaluated AFLC's computation of storage costs and concluded that the one percent rate was representative of actual storage costs (2:47-48). Storage costs represent the out-of-pocket costs that accrue in the keeping of an inventory and the amortized cost of the storage facilities (22:Encl.4,p.2). In a retention situation, storage costs are incurred if an item is retained in inventory and should be included as part of inventory holding costs.

The annual interest costs related to the funds committed to inventories have been assessed at ten percent of an item's purchase price (2:47;22:Encl.4,p.3). In a seminar conducted by AFLC, Dr. Jacob Stockfish (1:4) concluded that the ten percent rate was reasonable for use with government inventory investment. Each dollar of public funds which is invested in an Air Force inventory represents a dollar of investment in the private sector which is foregone (22:Encl.4,p.1). The interest cost included as a component of holding cost represents the opportunity cost of investments in inventory assets (6:13). Interest costs are an important consideration at the time assets are to be acquired since a measure of opportunity cost should be included in the decision to purchase inventory assets. A

decision to retain an item in inventory does not require an additional investment of public funds for purchase of inventory assets. However, the decision to retain an item in inventory precludes the salvage of that item and incurs an opportunity cost associated with the salvage value of the item. Interest costs, therefore, appear to be a necessary component of the cost of holding an item in inventory.

Obsolescence costs have also been assessed at a fixed annual percentage of an item's purchase price (22:Encl.4,p.2). AFLC currently uses a 13 percent annual obsolescence charge for all expendable items (2:48;4:108). The 13 percent rate is derived from averaging historical data, aggregated for all expendable items, on obsolescence, losses and deterioration, and quantities of items disposed of as excesses.

Obsolescence cost is charged for:

losses of materiel due to all causes that render the on-hand materiel superfluous to need. Thus this element will include losses due to technological obsolescence, over-forecasting of requirements, deterioration beyond the point of use, and other causes [22:Encl.4,p.1].

The obsolescence loss rate is computed by dividing the dollar value of transfers to Property Disposal Officers by the total dollar value of on-hand and on-order inventory assets. Application of this rate to holding costs is an attempt to avoid purchasing inventory items that will be disposed later (1:1). In a retention situation, the decision involves retaining inventory items that have already been purchased rather than purchasing new items.

However, the salvage value which can be obtained from an item is lost if the item is retained and subsequently becomes obsolete. Therefore, the obsolescence rate appears to be applicable to the salvage value of an item rather than to its purchase price.

Reprocurement Costs

If an item is disposed, a net salvage value equal to item salvage value minus disposal cost is realized. At a future date, additional items must be purchased to meet continuing demand for the item. Purchase costs include the cost of the items procured and the cost to order. The following paragraphs describe these three components of reprocurement costs--order cost, item cost, and net salvage value.

DODI 4140.39 (22:Encl.3) specifies that order costs are dependent on the type of procurement method used to obtain an inventory item. Separate order costs are required for small purchases (purchase value less than or equal to \$2500), large purchases (purchase value greater than \$2500), and call-type contracts. The instruction provides formulas for computing variable costs such as direct labor, indirect labor, and other administrative processes associated with the procurement function. The final cost obtained from the formulas represents the variable cost-to-order per item of inventory as opposed to the variable cost-to-order per contract.

The instruction requires each DoD agency to develop a technique for determining which cost-to-order category to use for each inventory item. AFLC uses dollar value of annual demand as an indicator of the procurement method to be used, as suggested in the instruction. Using historical data obtained from procurement personnel, AFLC has established a breakpoint figure of \$4900 of annual demand for determining the order cost value to be used for each item in the D062 System. An order cost of \$149 is applied to items having an annual dollar demand of \$4900 or less. An order cost of \$444 is applied to items having an annual dollar demand greater than \$4900 (3). Annual demand in dollars is obtained by multiplying annual demand rate times item cost.

Item cost, salvage value, disposal cost, and order cost are often assumed to remain constant over a period of analysis. However, these amounts generally change over time due to various factors. Since these values generally increase over time, the term inflation is often used to describe the cumulative effect of the various factors. For the purpose of this study, such a broad definition of the term inflation seems adequate.

The assessment of reprocurement costs should logically account for the effect of inflation on the amounts mentioned above. Since holding costs are computed on the basis of a percentage of item cost, inflation also affects holding costs and makes the inclusion of inflation

in an analysis of economic retention less than straight-forward. However, due to the effect of inflation on cost based decisions, it should be addressed as an economic consideration in relation to both holding and reprocurement costs.

Summary of Economic Criteria

Cost of holding and cost of reprocurement were established as appropriate criteria upon which to develop an economic retention model. Components of holding costs were found to be storage costs, interest costs, and obsolescence costs. Components of reprocurement costs were found to be the cost of placing an order, the cost of purchasing the required quantity of items, and the negative cost associated with the net salvage value obtained from disposed items.

ADDITIONAL CRITERION

Achieving the research objective of developing an economic retention level model for use by AFLC required explicit consideration of AFLC requirements in the development process. Interviews with personnel at AFLC/MMR (9) disclosed the following information.

1. AFLC needed a retention level model which could be integrated into the current D062 System. The ideal model would use data currently included in the system to preclude the addition of data elements for each of the 400,000 items maintained in the D062 System.

2. Real-time access to the model was not needed. It was felt sufficient to have the economic retention level computed quarterly and printed out on the same documents as it now appears.
3. The economic retention computations should be automated to the maximum extent possible to relieve the item manager of the need to manually perform the computations (15:1).

Based on the above information, an additional criterion was added:

The model must be easily integrated into the D062 System, using existing data and producing results which fit current output formats.

CHAPTER III

EXISTING RETENTION MODELS

In order to accomplish the objective stated in Chapter I, a literature search was conducted to determine the extent of retention level model development throughout DoD. A literature review indicated the existence of retention level models within the U. S. Army, the U. S. Navy, and the U. S. Air Force.

U. S. ARMY RETENTION MODEL

The U. S. Army's retention model is based on a study completed by Kaplan in 1969 (8). Kaplan's model is shown below with the terms defined as follows:

- S = storage cost expressed as a percentage of item purchase value.
- d = probable rate of deterioration.
- θ = probable rate of obsolescence.
- L = probable rate of physical loss.
- i = interest rate for discounting costs and benefits to present value.
- D = salvage value expressed as a percentage of item acquisition cost.
- t = time period over which an item is maintained in inventory.

$$D \leq \frac{(1 - t \cdot d) (1 - t \cdot \theta) (1 - L)^t}{(1 + i)^t}$$

$$- S \sum_{j=1}^t \frac{(1 - L)^j (1 - j \cdot \theta)}{(1 + i)^j}$$

Essentially, Kaplan's model as stated above uses the expression $\frac{(1 - t \cdot d) (1 - t \cdot \theta) (1 - L)^t}{(1 + i)^t}$ to discount

the probability that an item is not lost, obsolesced, and does not deteriorate during time period t . The expression

$$S \sum_{j=1}^t \frac{(1 - L)^j (1 - j \cdot \theta)}{(1 + i)^j} \text{ reflects the discounted storage}$$

cost percentage associated with items that are not lost and are not obsolesced during time period t (8:Chap.1). By subtracting the latter percentage figure, the Army obtains the cost of maintaining an item in inventory expressed as a percentage of item purchase value. The other side of the equation, the salvage value D , is also expressed as a percentage of an item's purchase value. By solving the equation for t , an economic retention level in years may be found at which the cost to hold an item in inventory is equal to the disposal value of the item. By using current demand rate information, the period t may be converted to item units which represent the Army's economic retention level for the item under consideration.

The Army has provided the Kaplan model for use at depot level by item managers. A table has been generated

from which the item manager selects an item's retention level in terms of years by entering the table with percentage values for deterioration, obsolescence, loss, and storage rates (18:28). Normally, the Army assigns values of five, five, two and one percent to d, θ , L, and S, respectively, yielding a retention level of 8.8 years when applied against a ten percent salvage value. However, the table is constructed so that other values of the input parameters may be examined for items with special characteristics. After obtaining the retention level in terms of years, the item manager's procedure for determining the amount of stock to hold is to multiply the number of years obtained from the table by the current annual demand rate (18:31).

In support of the Kaplan model employed by the Army, a study completed by the Logistics Management Institute in 1971 stated that "The Army is currently using an excellent method for determining ERL at the WICP [depot] level [18:26]." However, adaptation of the model for AFLC inventory management would appear to have two disadvantages. First, the model's input factors require review by the item manager. It has been stated that the thrust of the effort to solve the economic retention problem should be toward relieving the item manager of this task. By developing retention level determination techniques usable in conjunction with centrally located inventory management information systems, item managers could then expend more of their time

in closer management of critical, high value items (16:1). If the thrust of problem solution is to be in this direction, it would appear that the model's reliance on an item manager's knowledge is a departure from the desired course of action.

A second disadvantage of the model stems from input factor definitions. Loss, obsolescence, and deterioration are treated separately in the Kaplan model. However, as discussed in the previous chapter, the Air Force uses a single rate of 13 percent to cover these three factors. Substitution of the 13 percent rate for Kaplan's obsolescence, deterioration and loss components would be feasible if all three components were included in each part of the model. A review of Kaplan's model shows that this is not the case since storage costs are only adjusted for the probability of loss and obsolescence. Clearly, to substitute the Air Force's 13 percent summary rate for only two of the three factors would constitute a violation of the model's logic developed in Kaplan's paper (8:4).

In summarizing the Kaplan model currently used by the Army, advantages and disadvantages of adaptation for Air Force use are in evidence. While the model is considered excellent in addressing holding, disposal and repurchase costs of an item, when needed, its suitability for use with current management information systems appears limited. Additionally, there appears to be an incompatibility in treatment of holding cost component

factors between services which would limit the model's adaptability for AFLC use.

U. S. NAVY RETENTION MODEL

Item managers at U. S. Navy supply depots have used an economic retention model since 1960. A derivation of the model was not located in the literature. However, through analysis of the stated model, a suggested derivation is presented below.

The input terms are defined as follows:

- U = unit purchase price.
- C = cost to order a quantity of items for placement in inventory. C is expressed as dollars per order.
- F_1 = the apportionment year strength factor, a dimensionless adjustment factor relating the past year's demand experience with the anticipated demand experience for the period over which retention of items is being considered. If an increase in demand is anticipated, the factor will be greater than one. For identical demand experience the factor is equal to one.
- I = time interval between placement of orders. I is expressed in years.
- i = the interest rate applicable to funds committed to inventories. i is in terms of dollars per dollar per year.
- P = the ratio of salvage value to purchase cost of an item.
- s = storage cost expressed as dollars per dollar per year.
- D = annual demand for an inventory item. D is expressed in units per year.

b = probability of obsolescence expressed as the reciprocal of the expected life of an item in years. Therefore, the units associated with $b = 1/\text{years}$.

ERR = Economic Retention Requirement expressed in terms of years of demand (24:Encl.2,p.1).

If a quantity of items above computed requirements is disposed, an equal quantity must be ordered at some future time to meet the demand for that item. The cost of disposal can be stated as:

cost to order = order cost + item cost - net salvage value of disposed items.

cost to order = $C + F_1 \text{IDU} - P (F_1 \text{IDU})$

If a quantity of items above the computed requirements is retained, inventory holding costs are incurred.

cost to hold = storage cost + obsolescence cost of items retained + cost to order items to replace those lost to obsolescence + opportunity cost of salvage value foregone by retaining the items.

cost to hold = $s (F_1 \text{IDU}) + b (F_1 \text{IDU}) + C_b + P_i (F_1 \text{IDU})$

The point at which cost to order equals cost to hold determines the break-even point referred to in BUSANDAINST 4440.38A (24:1). If the cost to order is greater than the cost to hold, then it is cheaper to retain a quantity of items in inventory. Defining the Economic Retention Requirement (ERR) as a ratio of cost to order to cost to hold provides a measure of the retention requirement in terms of years of demand. Algebraically:

$$ERR = \frac{C + F_1 IDU - P F_1 IDU}{s F_1 IDU + b F_1 IDU + P i F_1 IDU + C b}$$

$$= \frac{C + (1 - P) (F_1 IDU)}{(s + b + P i) (F_1 IDU) + C b}$$

$$= \frac{1 - P + \frac{C}{F_1 IDU}}{b + \frac{C b}{F_1 IDU} + s + P i}$$

(24:Encl.2,p.1).

To determine the item quantity which should be maintained in inventory, the annual demand rate should be multiplied by the ERR.

The Navy's Economic Retention Requirement model is used at depot level. Referring to the implementing directive, item managers are required to develop an obsolescence probability (b), an apportionment year strength factor (F_1), and a ratio of salvage value to unit purchase price (P) for each item managed. Additionally, the directive contains separate tables of computed ERR values for obsolescence probabilities ranging from ten to 33 percent (24:Encl.2,p.1).

Use of the Navy's model by the Air Force would appear to have disadvantages similar to the ones previously discussed in relation to the Army's model. The item manager must manually determine values of the input cost factors. As previously stated, this appears to disagree with the trend of current policy (16:1). In contrast to the Army's model, losses and deterioration could be

included in the obsolescence rate b. AFLC's 13 percent obsolescence rate could be substituted into the Navy's model without the difficulty encountered with the Army's model. However, AFLC has not taken a demand forecasting approach which uses a factor similar to the apportionment year strength factor F_1 . The cost and time required to develop such a factor for use by AFLC in an adapted version of the Navy model is unknown at the present time (9). Although these disadvantages are not insurmountable, current policy and consistency of cost factor uses and definitions seem to preclude use of the U. S. Navy model by AFLC.

U. S. AIR FORCE RETENTION MODEL

Writing for AFLC in 1960, Irving Katz produced one of the first studies on economic retention (10). However, his work and resulting model addressed only reparable items which are beyond the scope of this research. Adaptability of the model to expendable EOQ type items addressed in this study is limited due to Katz's treatment of repair cycles which are not applicable to expendable items. Nevertheless, Katz's work appears to have made a significant contribution to the field of economic retention since it seems to have provided basic ideas for Kaplan's economic retention model currently used by the Army.

SUMMARY

The economic retention models used by the Army and the Navy have been presented in this chapter. Additionally, disadvantages with regard to implementation of these models by AFLC have been discussed. In view of the stated disadvantages, an economic retention model which addresses AFLC's existing information system and directives regarding cost factors will be developed in the next chapter.

CHAPTER IV

MODEL DEVELOPMENT

In the previous chapter, the economic retention models used by the Army and the Navy were analyzed with regard to suitability for adaptation to AFLC's present operating environment and policy constraints. Although both models appear to be developed around widely accepted inventory management theory, disadvantages of adaptation were cited for both models. Therefore, in this chapter, an alternate model will be proposed.

The variables in the proposed model are defined where they are introduced within this chapter. A complete listing of variable definitions used in this chapter and in Chapter V is contained in Appendix A.

BASIC MODEL

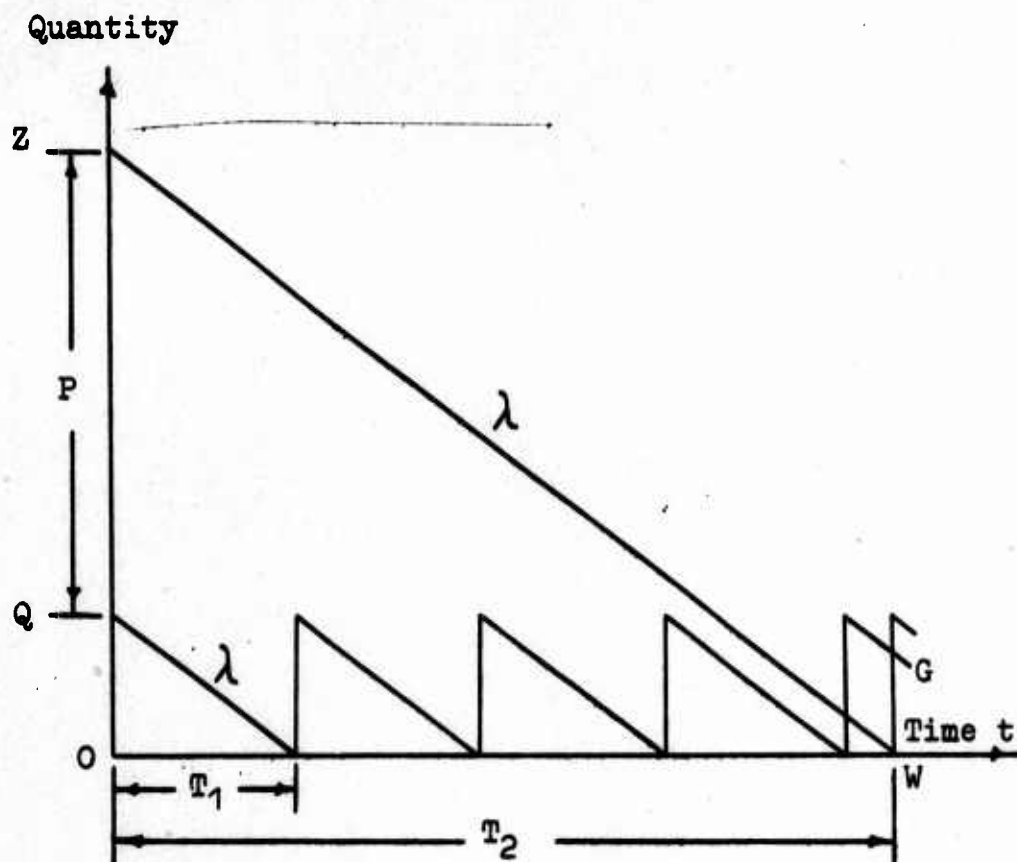
Assuming that the demand rate for an item remains relatively stable (23:Encl.2,p.2), two options are available to the item manager faced with a retention decision. First, he can retain a quantity of items in inventory to meet the continuing demand. Under this option, the next EOQ purchase would not be made until the normal reorder point is reached. Second, he can dispose of an item and continue purchasing additional items in accordance

with the normal EOQ timetable. The effect of retaining a quantity of items above the EOQ level can be evaluated by examining the foundation of the basic EOQ model. Figure 2 represents the behavior of on-hand inventory over time under normal EOQ operation as a saw-tooth curve running from point Q to point G. The retention of a quantity P above the requirement level Q is shown by line ZW, where $Z = Q + P$. If the quantity P is held in inventory, additional holding costs are incurred over the time period T_2 . The extra cost of holding these items can be determined by multiplying the inventory holding cost factor times the item cost times the average level of inventory above the normal inventory level. The increase in average inventory level can be found by subtracting the area under the saw-tooth curve between the origin O and point W from the area of the triangle OZW. The result can be expressed as:

$$\begin{aligned}
 \text{Net Area} &= \int_0^{T_2} (Z - \lambda t) dt - \frac{T_2}{T_1} \int_0^{T_1} (Q - \lambda t) dt \\
 &= \left[Zt - \frac{\lambda t^2}{2} \right]_0^{T_2} - \frac{T_2}{T_1} \left[Qt - \frac{\lambda t^2}{2} \right]_0^{T_1} \\
 &= \left(ZT_2 - \frac{\lambda T_2^2}{2} \right) - \frac{T_2}{T_1} \left(QT_1 - \frac{\lambda T_1^2}{2} \right)
 \end{aligned}$$

Since $Z = \lambda T_2$ and $Q = \lambda T_1$, the above expression

becomes:



$$Q = \sqrt{\frac{2A\lambda}{IC}} \quad \text{Economic order quantity computed by AF EOQ model (4:38)}$$

λ = annual demand rate

T_1 = normal time between orders (Q/λ)

T_2 = economic retention period ($(Q + P)/\lambda$)

P = economic retention quantity ($Z - Q$)

Figure 2

Inventory Position Over Time For Retention
And Disposal Alternatives

$$\begin{aligned}\text{Net Area} &= T_2 \left(Z - \frac{Z}{2} \right) - \frac{T_2}{T_1} \left[T_1 \left(Q - \frac{Q}{2} \right) \right] \\ &= \frac{T_2 Z}{2} - \frac{T_2 Q}{2}\end{aligned}$$

But, $Z = Q + P$, so:

$$\text{Net Area} = \frac{T_2}{2} \left(Q + P \right) - \frac{T_2 Q}{2} = \frac{T_2 P}{2}$$

The increase in the average inventory level over the time period T_2 is:

$$T_2 \frac{P}{2} \cdot \frac{1}{T_2} = \frac{P}{2} \quad (13).$$

The increase in inventory holding cost for the period T_2 is computed by multiplying the cost of holding one item in inventory for T_2 years times the average inventory level.

The annual holding cost factor I is multiplied by T_2 years to obtain the holding cost factor over the T_2 period.

$$\text{Holding cost} = (IT_2 C) \frac{P}{2}$$

If the quantity P is disposed, P units are ordered during the period T_2 . The cost of ordering includes the cost of the P units and the cost of placing the number of orders required to obtain the P units. Since Q units are purchased each time an order is placed, the number of orders required to purchase P units is $\frac{P}{Q}$. If A is defined as the cost of placing an order and C is the item cost, then total order costs are given by the equation:

$$\text{Total order cost} = CP + \frac{AP}{Q} .$$

If the P items are disposed, then total costs are reduced by an amount $P(S - D)$ where:

S = salvage value per item.

D = disposal cost per item.

If the P items are retained, $P(S - D)$ represents an amount that is not available for investment elsewhere. This amount is an opportunity cost and increases the total cost of retaining the P items by an amount $P(S - D)JT_2$, where J is the annual charge associated with an opportunity cost. Multiplication by T_2 provides the opportunity cost over the period T_2 .

Equating cost of holding and cost of ordering and solving for T_2 should give the time period over which it is as economical to retain P items as to dispose and reprocur / them at a future date.

$$(1) \quad T_2 \left(\frac{ICP}{2} + PJ(S - D) \right) = CP + \frac{AP}{Q} - P(S - D)$$

Rearranging terms gives the following equation in terms of T_2 :

$$T_2 = \frac{P \left(C + \frac{A}{Q} - (S - D) \right)}{P \left(\frac{IC}{2} + J(S - D) \right)}$$

Dividing the numerator and the denominator of the right hand expression by C yields:

$$T_2 = \frac{1 + \frac{A}{CQ} - \frac{(S - D)}{C}}{\frac{1}{2} + \frac{J(S - D)}{C}}$$

Expressing S and D as percentages of item cost, $S = KC$ and $D = LC$ (K and L are percentages), gives the following equation:

$$(2) \quad T_2 = \frac{1 + \frac{A}{CQ} - (K - L)}{\frac{1}{2} + J(K - L)}$$

To convert the years of demand T_2 to the economic retention quantity P, recall from Figure 2 that

$$(3) \quad \begin{aligned} P &= Z - Q \\ P &= \lambda T_2 - Q \end{aligned}$$

P represents the quantity which can be held in inventory for the same costs as would be incurred by disposing of that quantity and repurchasing later. As such, it represents a maximum economic retention level and can be computed for each item in the inventory. P can be expressed in terms of years of demand consistent with current AFLC practice by using the value of T_2 as found in Equation (2).

The development of the model as given by Equation (2) does not account for the timing of the cash flows for payments of inventory holding costs and order costs. Implicit in Equation (2) is an assumption that inventory holding costs and order costs for the T_2 period are paid as lump sums. In actual practice, these payments are made at various times during the T_2 period. The impact of

inflation also has not been included in Equation (2). The remainder of this chapter will be devoted to modifying the model to include inflation and discounting.

BASIC MODEL WITH INFLATION

The impact of economic inflation on the model would appear limited assuming equal escalation in costs for labor and new materials. However, the return on disposed goods could experience a different rate of inflation than the rate related to costs. Accounting for this difference is relatively easy by establishing the following values, if known and considered significant by the user of the model.

Let R_1 = the cost inflation rate.

R_2 = the salvage value inflation rate.

Each cost factor should then be multiplied by $(1 + R_1)$ to obtain the adjusted cost due to inflation. Similarly, the net salvage factors on both sides of the equation should be multiplied by $(1 + R_2)$. For simplification, let

$(1 + R_1) = X_1$ and $(1 + R_2) = X_2$. Equation (1) then becomes

$$\frac{ICPT_2 X_1}{2} + PJT_2 (SX_2 - DX_1) = CPX_1 +$$

$$\frac{AP}{Q} X_1 - P (SX_2 - DX_1)$$

Dividing both sides of the equation by X_1 yields the following:

$$\frac{ICPT_2}{2} + PJT_2 \left(S \frac{X_2}{X_1} - D \right) = CP + \frac{AP}{Q} - P \left(S \frac{X_2}{X_1} - D \right)$$

Solving for T_2 in the manner previously described shows that:

$$T_2 = \frac{1 + \frac{A}{CQ} - \left(\frac{SX_2}{CX_1} - \frac{D}{C} \right)}{\frac{1}{2} + J \left(\frac{SX_2}{CX_1} - \frac{D}{C} \right)}$$

Letting $K = \frac{S}{C}$ and $L = \frac{D}{C}$, the above equation becomes:

$$(4) \quad T_2 = \frac{1 + \frac{A}{CQ} - \left(K \frac{X_2}{X_1} - L \right)}{\frac{1}{2} + J \left(K \frac{X_2}{X_1} - L \right)}$$

The effect of inflation on the model can be accounted for by substituting the following expression for K into Equation (4):

$$K' = K \frac{X_2}{X_1}$$

Consideration will now be given to the effect of discounting on the economic retention level model.

DISCOUNTED MODEL

The purpose of this section is to modify the model given by Equation (1) to include discounting.

Total cost of holding the retention quantity P for the period T_2 is given by the equation:

$$T_2 \left(\frac{ICP}{2} + PJ(S - D) \right) = CP + \frac{AP}{Q} - P(S - D)$$

Assuming that the cost of holding is assessed at the end of each year for the average inventory level on hand during that year, the following equations result:

Holding cost at the end of the first year--

$$\frac{ICP}{2} + PJ (S - D)$$

Holding cost at the end of the second year--

$$\frac{IC}{2} (P - \lambda) + (P - \lambda) (J) (S - D)$$

Holding cost at the end of the T_2 year--

$$\frac{IC}{2} (P - \lambda(T_2 - 1)) + (P - \lambda(T_2 - 1)) (J) (S - D)$$

Using the present value factor $\frac{1}{(1+i)^n}$ to discount each

cash flow gives the following expression for the present value of the annual payments for holding costs over the retention period T_2 :

Present value of holding cost =

$$(5) \quad \sum_{n=1}^h \left[\frac{IC}{2} (P - \lambda(n-1)) + J (S - D) \cdot (P - \lambda(n-1)) \right] \left[(P/F, i\%, n) \right]$$

where $(P/F, i\%, n)$ denotes the single payment present value factor (5:34) and h is the largest integer smaller than T_2 .

For the case where no items are retained, the normal EOQ quantities will be purchased according to the EOQ year timetable. The total cost of ordering P items over the time period T_2 is given by the equation:

$$\text{Order cost} = CP + \frac{AP}{Q} - P (S - D)$$

Actual cash flows resulting from the EOQ purchases will occur throughout the time period T_2 . Net salvage,

$P (S - D)$, is a one-time cash flow which occurs at the time of disposal. Cash flows for the EOQ orders will occur every T_1 years and be of the magnitude $(CQ + A)$. Discounting these cash flows results in the expression:

Present value of order cost--

$$(6) \quad - P (S - D) + \sum_{n=1}^m (CQ + A) (P/F, i\%, nT_1)$$

where m = the integer portion of $\frac{T_2}{T_1}$.

Equating the present values gives the following retention level model which includes discounting:

$$(7) \quad \sum_{n=1}^h \left[\frac{IC}{2} (P - \lambda(n-1)) + J (S - D) (P - \lambda(n-1)) \right]$$

$$\left[(P/F, i\%, n) \right] = - P (S - D) + \sum_{n=1}^m (CQ + A) (P/F, i\%, nT_1)$$

Equation (7) does not simplify algebraically because of the changes in holding costs from year to year and because the present value factors applied to holding costs and order costs are of different magnitudes. Since the limits on the summations and the unknown quantity P in Equation (7) are both related to T_2 , an iterative procedure can be used to solve for P . Noting that h is the largest integer less than T_2 , m is the integer portion of

$\frac{T_2}{T_1}$, and $P = \lambda T_2 - Q$ (from Figure 2) Equation (7) becomes

$$\begin{aligned}
 (8) \quad & \sum_{n=1}^h \left[\frac{IC}{2} \left(\lambda T_2 - Q - \lambda(n-1) \right) + J(S-D) \right. \\
 & \left. \left(\lambda T_2 - Q - \lambda(n-1) \right) \right] \left[(P/F, i\%, n) \right] \\
 & + P(S-D) - \sum_{n=1}^m (CQ + A) (P/F, i\%, nT_1) = 0
 \end{aligned}$$

The iterative solution procedure involves substituting successively larger values of T_2 into Equation (8) until the equation is satisfied. P is found by substituting the value of T_2 which satisfies Equation (8) into the equation $P = \lambda T_2 - Q$. The retention quantity in terms of years of demand is represented by T_2 .

Equation (8) was evaluated by use of the computer program in Appendix B for the following parameter values:

$$\begin{aligned}
 i &= .10 \\
 I &= .01 \\
 J &= .23 \\
 S &= .1C \\
 D &= 0 \\
 A &= \$149 \text{ or } \$444
 \end{aligned}$$

$$Q = \sqrt{\frac{2A\lambda}{(I+J)C}}$$

$$\$1 \leq C \leq \$10,000$$

$$1 \leq \lambda \leq 200,000$$

The values of A given above are the values which AFLC currently uses for order cost as explained in Chapter II.

The retention period T_2 found for the above values ranged from 28 to 82 years. Consideration of cash flows and the time value of money completes the development of the economic retention model.

SUMMARY

An economic retention model based on holding costs and order costs was developed in this chapter. A preliminary model, Equation (2), was derived to illustrate the basic relationships involved in a retention situation. A final model, Equation (8), was derived to include the effects of discounting cash flows for holding costs and order costs. The following chapter will examine the sensitivity of the economic retention model of Equation (8) to changes in its input parameters.

CHAPTER V

SENSITIVITY ANALYSIS

In previous chapters, the parameters of a proposed economic retention model were established and the values currently used for those parameters by AFLC were given. In this chapter, the sensitivity of the model to its parameters will be examined. The steps followed in conducting the sensitivity analysis are:

1. Identify appropriate ranges of values for each parameter.
2. Examine the sensitivity of the model by varying each cost factor over its applicable range while holding the remaining parameters constant.

INPUT FACTORS

Table 1 shows the range of values used in the sensitivity analysis for each parameter. With the exception of order cost, the ranges were arbitrarily established but included the highest and lowest values of each parameter found in the literature reviewed in Chapter II. The value used for order cost was based on the dollars of annual demand for an item as described in Chapter II.

Table 1
Ranges of Values For Model Parameters

Parameter	Variable Name	Range
Order Cost	A	\$149 or \$444
Item Cost	C	\$1 to \$10,000
Annual Demand	λ	1 to 200,000 items
Storage Cost	I	1 - 30 percent
Opportunity Cost (Interest + Obsolescence)	J	10 - 40 percent
Net Salvage	(K - L)	5 - 50 percent
Cost Inflation	R_1	0 - 15 percent
Salvage Inflation	R_2	0 - 15 percent

MODEL SENSITIVITY

The purpose of the sensitivity analysis was to determine the change in retention years Y resulting from changes in each of the model parameters: A , C , λ , I , J , $(K - L)$, R_1 , and R_2 . A comparison of the percent change in Y for a given percent change in each model parameter was made to determine the sensitivity of the model to each parameter relative to the other parameters.

Equation (8) was used to analyze the sensitivity of the model to changes in each parameter. The computer program contained in Appendix B was used to conduct the analysis. By varying each parameter through the range identified in Table 1 and holding the remaining parameters at the values currently used by AFLC, the effect on retention years was examined.

Order Cost (A), Item Cost (C), Annual Demand Rate (λ)

Annual demand in dollars is computed by multiplying item cost by annual demand rate. Accordingly, order cost is dependent upon the item cost and demand rate. For each of the two values of order cost, the effects of item cost and annual demand rate on retention years was determined by use of the computer program in Appendix B. The following values were used for the remaining parameters:

$$I = .01$$

$$J = .23$$

$$(K - L) = .1$$

$$R_1 = R_2 = 0$$

Item cost was varied from \$1 to \$4900 for annual dollar demands of \$1000, \$2000, \$3000, \$4000, and \$4900. For these runs of the program, order cost was \$149. Another set of runs was made in which item cost was varied from \$1 to \$10,000 for annual demands of \$5000, \$10,000, \$15,000, \$20,000, \$25,000, \$50,000, \$75,000, \$100,000, and \$200,000. An order cost of \$444 was used for these runs.

A graph of retention years versus item cost for selected values of annual demand was used to illustrate the results of the computer runs. Figure 3 shows the results for item costs between \$1 and \$1000 using annual dollar demands of \$1000, \$3000, and \$4900, where order cost is \$149. Table 2 shows the retention years for those same values of annual dollar demand for item cost ranging from \$1000 to the break point value of \$4900. Note that there was no change in retention years throughout this range. Since the change in retention years is greatest for item costs between \$1 and \$100, a graph of this range of item costs was also constructed and is shown in Figure 4.

Figure 5 shows the results for item cost between \$1 and \$1000 using annual dollar demands of \$5000, \$15,000

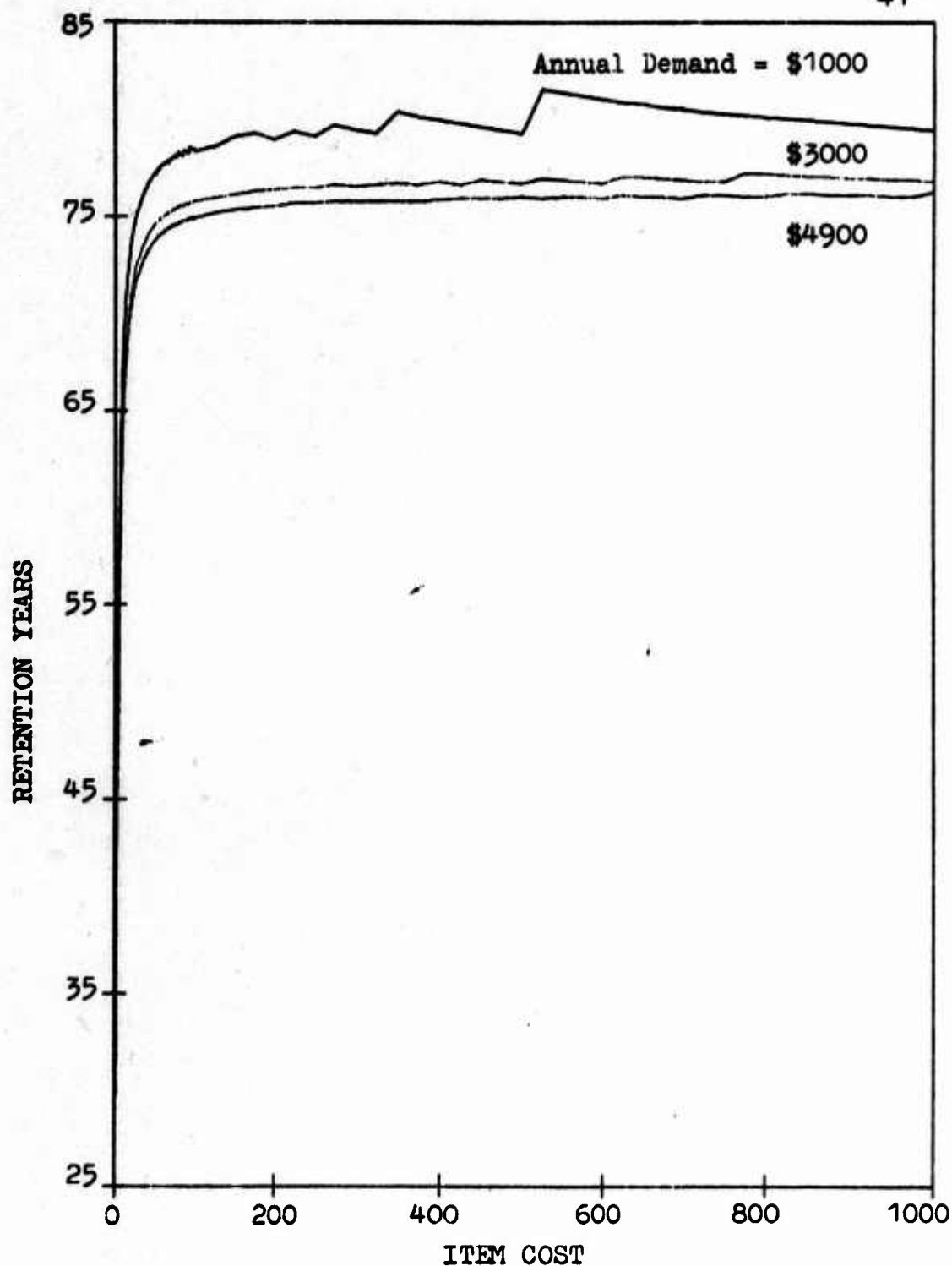


Figure 3

Retention Years Versus Item Cost For Annual
Demands of \$1000, \$3000, and \$4900
(Order Cost = \$149, $1 \leq C \leq 1000$)

Table 2
Retention Years For Selected Values of
Annual Dollar Demands For $C \geq \$1000$

Dollars of Annual Demand	$C = \$1000$	$C = \$ \text{Annual Demand}$
2000	78	78
3000	77	77
4000	76	76
4900	76	76

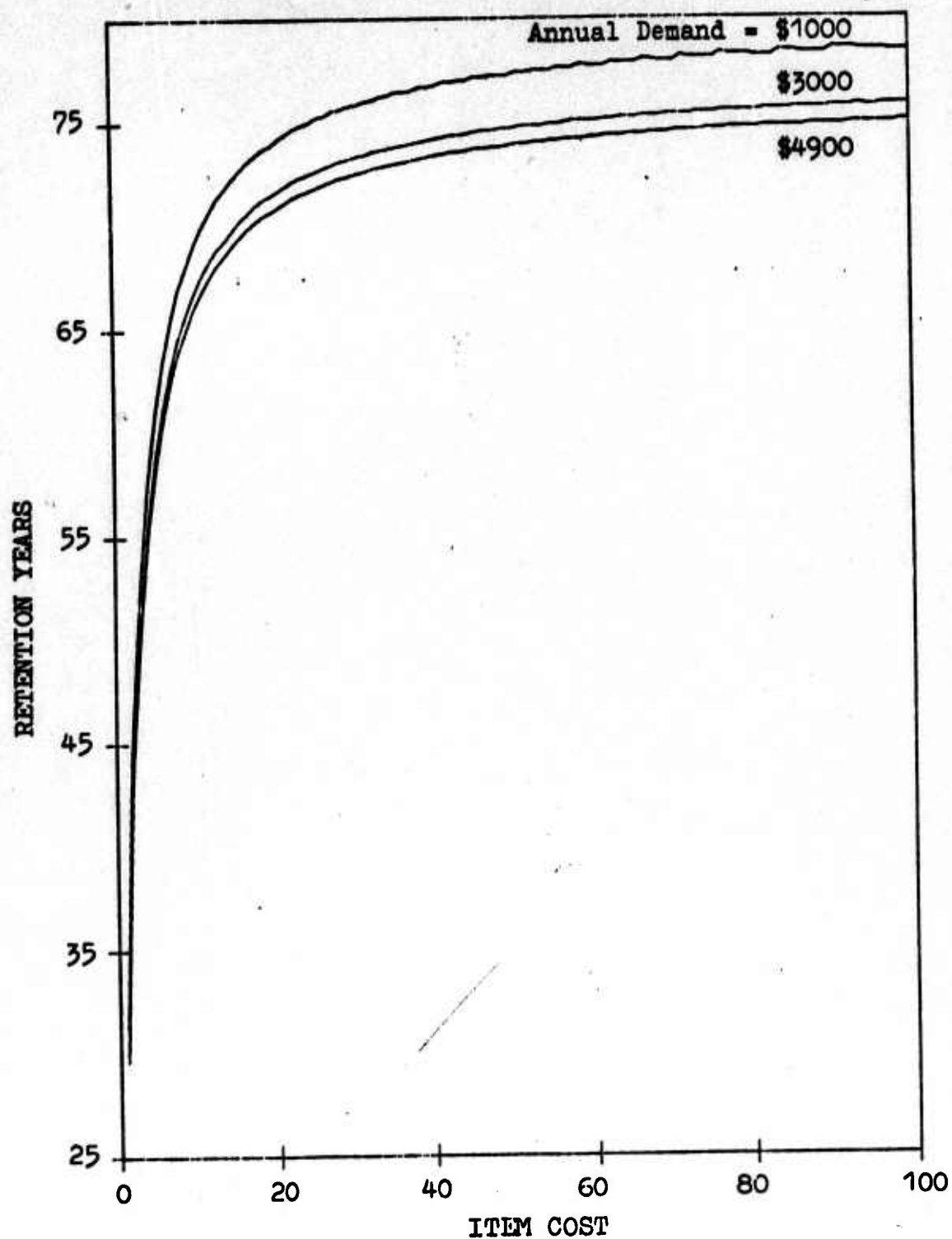


Figure 4

Retention Years Versus Item Cost For Annual
Demands of \$1000, \$3000, and \$4900
(Order Cost = \$149, $1 \leq C \leq 100$)

and \$200,000 and an order cost of \$444. Table 3 shows the retention years associated with these dollar demand values for item cost ranging from \$1000 to \$10,000. Note that the maximum change in retention years throughout this range was one year. Since the change in retention years was greatest for item costs between \$1 and \$100, a graph of this range of item costs is provided in Figure 6.

The sensitivity of the model to changes in order cost A was examined for 100 percent increases in the two values of A used by AFLC. For an annual dollar demand of \$3000, an increase in A from \$149 to \$298 increased retention years Y a maximum of three percent. For an annual dollar demand of \$15,000, an increase in A from \$444 to \$888 increased Y a maximum of two percent. These results indicated that the model was relatively insensitive to changes in A.

The sensitivity of the model to changes in item cost C and demand rate λ was determined by inspection of Figure 4. The graph shows that for a given annual demand, the rate of increase in Y decreased as C increased. The values in Table 4 were obtained from the computer runs for an annual demand of \$3000 to determine the percent change in Y for a given change in C. The results depicted in Table 4 indicated that the model was relatively sensitive to changes in C for values of C less than or equal to \$10 and relatively insensitive to changes in C for values of C greater than \$10. Figure 4 shows that the maximum change in retention years Y as demand rate λ was increased occurred at C = \$100. Since annual dollar demand was

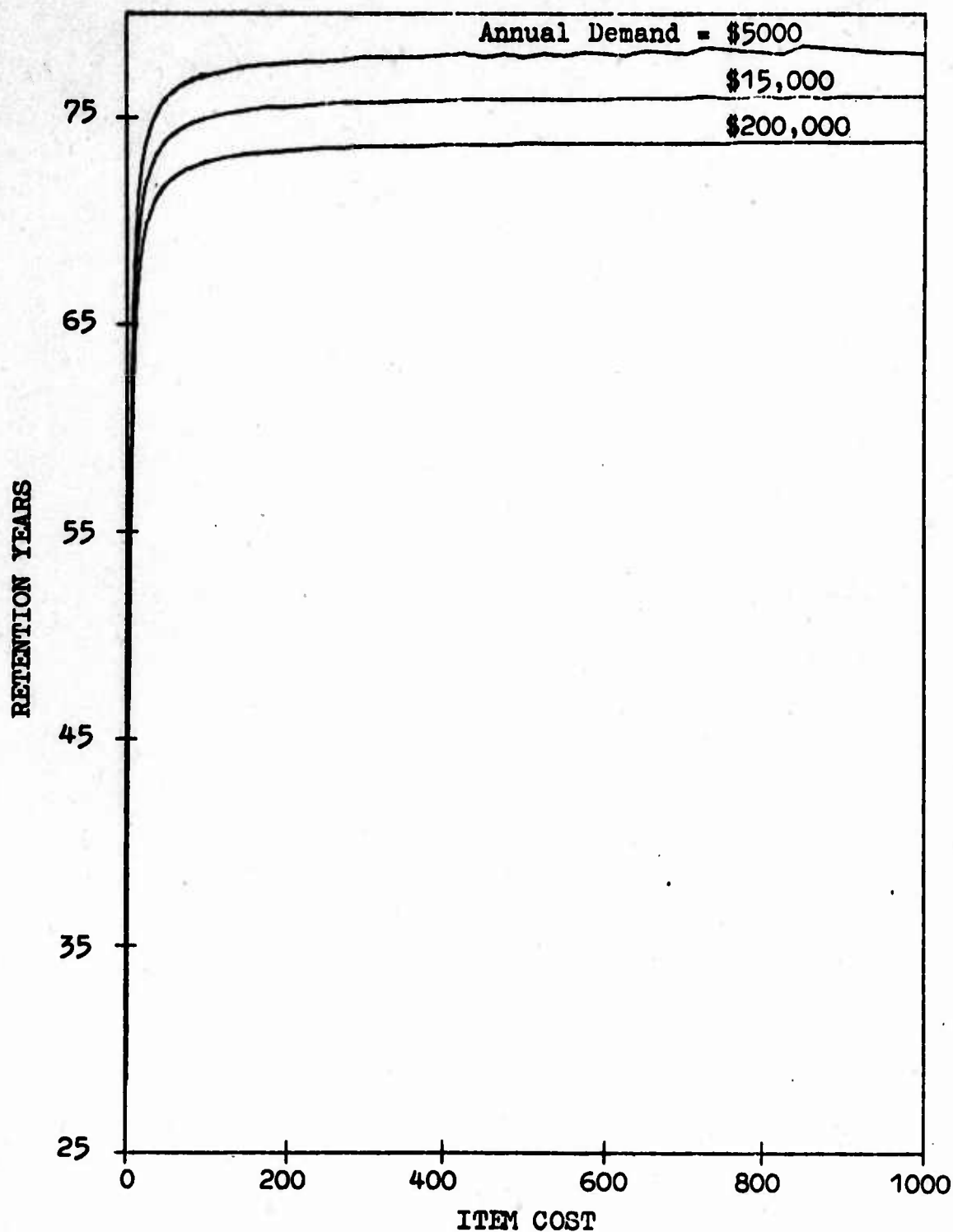


Figure 5

Retention Years Versus Item Cost For Annual
Demands of \$5000, \$15,000, and \$200,000
(Order Cost = \$444, $1 \leq C \leq 1000$)

Table 3

Retention Years For Selected Values of Annual
Dollar Demands For C = \$1000 and C = \$10,000

Dollars of Annual Demand	C = \$1000	C = \$10,000
5,000	78	78*
10,000	77	77
15,000	76	77
20,000	76	76
25,000	75	76
50,000	75	75
75,000	74	75
100,000	74	74
200,000	74	74

*C = \$5000, otherwise Annual Demand would not equal \$5000.

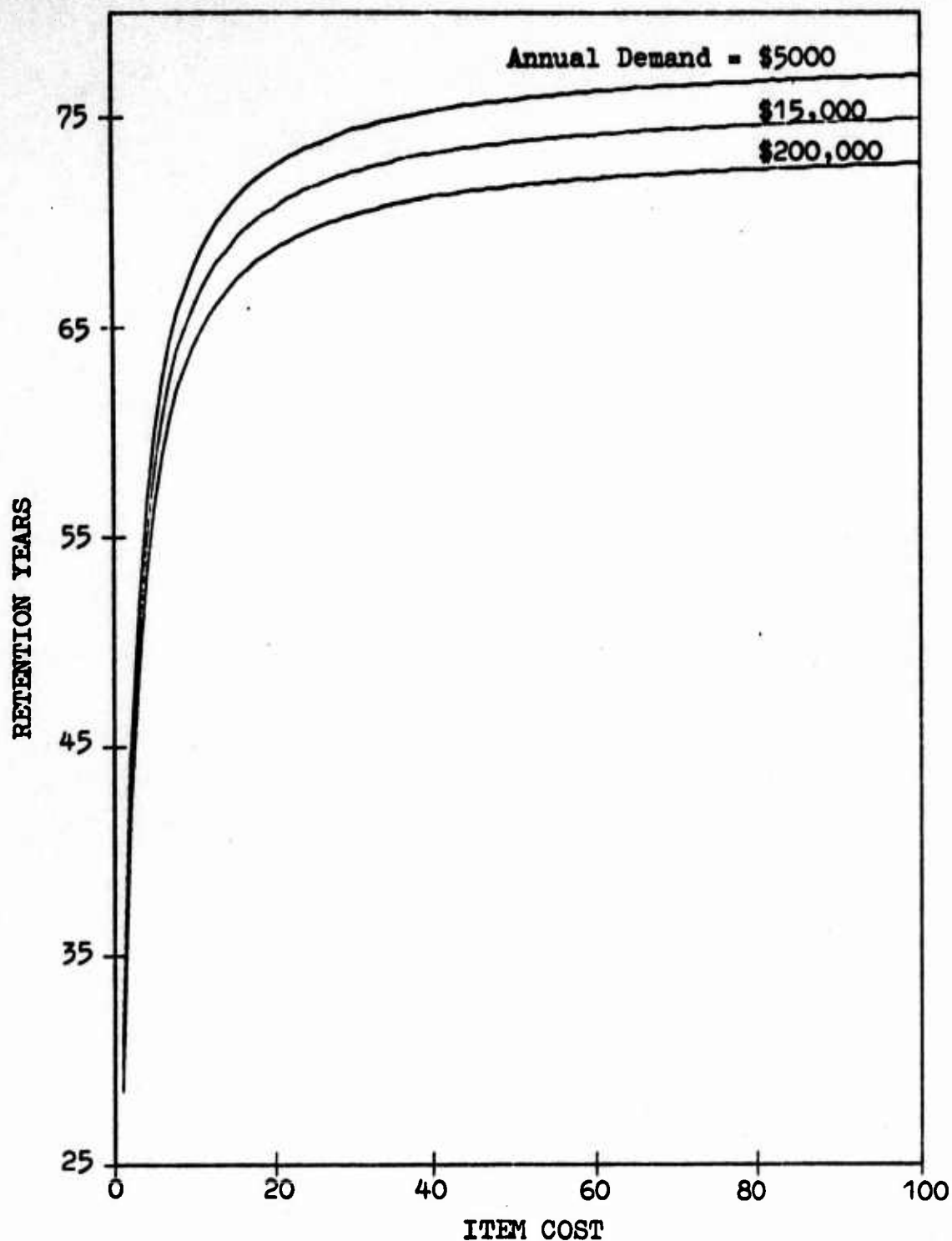


Figure 6

Retention Years Versus Item Cost For Annual
Demands of \$5000, \$15,000 and \$200,000
(Order Cost = \$444, $1 \leq C \leq 100$)

Table 4
Percent Change in Y For a 100 Percent Change
In C For Selected Values of C

C	Y	Percent Change in C	Percent Change in Y
1	30	100	43
2	43		
5	59	100	14
10	67		
50	75	100	1
100	76		

found by multiplying C times λ , an increase in annual demand from \$1000 to \$3000 corresponded to an increase in λ from ten to 30 units, a 200 percent change. Y changed from 78 years to 75 years, a four percent change. The results indicated that the model was relatively insensitive to changes in annual demand rate λ .

The similarity between Figure 4 and Figure 6 indicated that the same results described above for items having an annual dollar demand less than or equal to \$4900 applied to those items having an annual dollar demand greater than \$4900. Accordingly, the following sections will focus only on the \$3000 annual demand value to determine the percent change in Y for a given percent change in the remaining parameters. However, graphs of the computer runs for both \$3000 and \$15,000 annual demand were constructed.

Storage Cost Factor (I)

The sensitivity of the model to changes in I was examined using the following parameter values:

$$I = .01, .05, .10, .20, .30$$

$$J = .23$$

$$(K - L) = .1$$

$$R_1 = R_2 = 0$$

$$\$1 \leq C \leq \$100$$

$$\text{Annual Dollar Demand} = \$3000 \text{ and } \$15,000$$

Item cost above \$100 was not considered because of the small

change in retention years for an increase in item cost above \$100, as shown in Figure 3. However, within the range of \$1 to \$100, the selection of a \$3000 annual demand provided an indication of the change in retention years to be expected with the \$149 order cost. Selection of the \$15,000 annual demand provided a similar indication for the \$444 order cost as shown in Figure 5.

The results of the computer runs for the values of I stated above are shown in Figure 7 for an annual demand of \$3000. Figure 8 contains the results of the computer runs for an annual demand of \$15,000. For an item cost of \$100, Figure 7 shows that as the storage cost factor I increased from .10 to .20, retention years Y changed from 25 years to 15 years. A 100 percent change in I resulted in a 40 percent change in Y . These results indicated that the model was relatively sensitive to changes in I .

Opportunity Cost Factor (J)

The following parameter values were used to analyze sensitivity of the model to changes in J :

$$I = .01$$

$$J = .10, .23, .30, .40$$

$$(K - L) = .10$$

$$R_1 = R_2 = 0$$

$$\text{Annual Dollar Demand} = \$3000 \text{ and } \$15,000$$

The results of the analysis are shown in Figure 9 for an annual demand of \$3000 and in Figure 10 for an annual

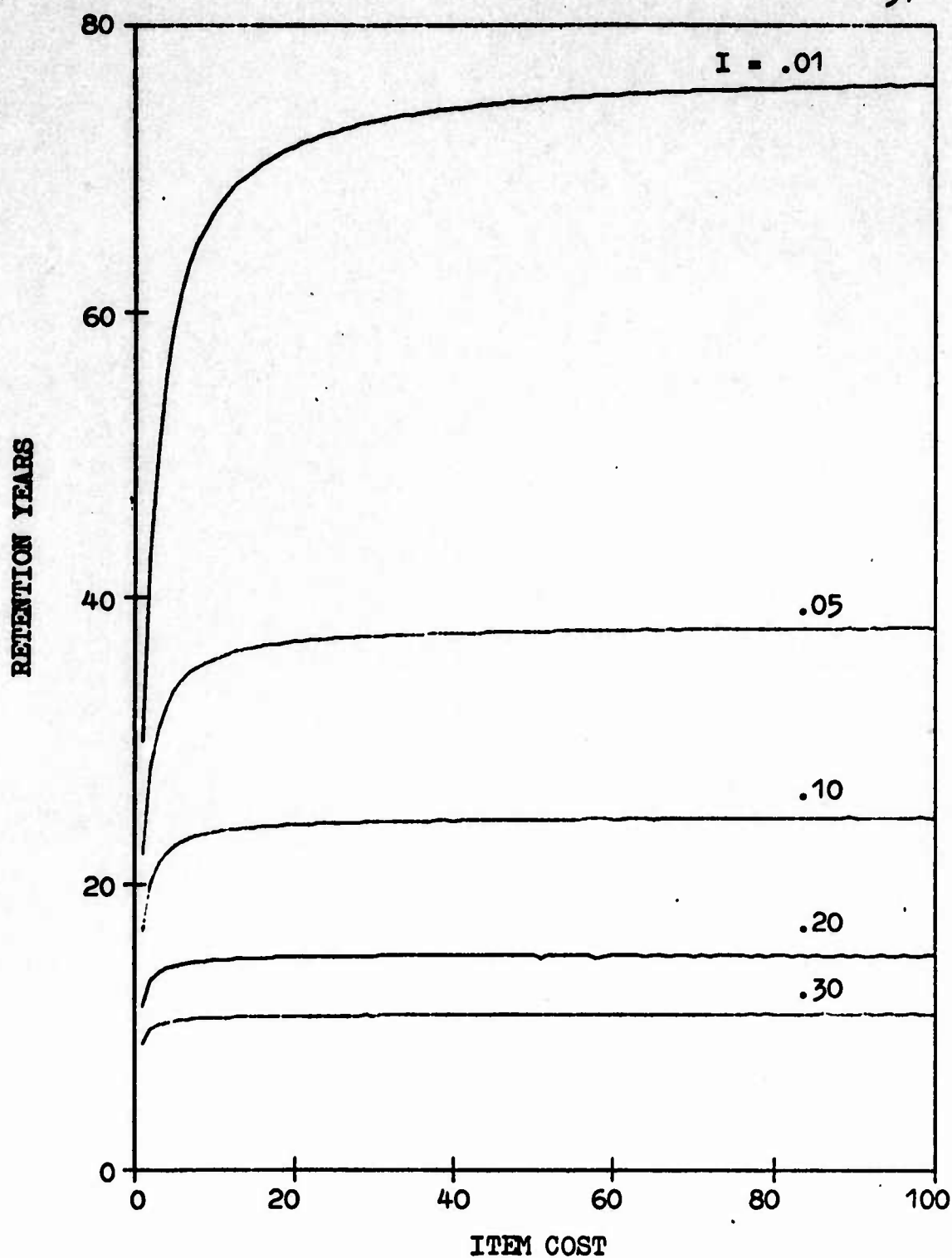


Figure 7

Retention Years Versus Item Cost For $I = .01, .05, .10, .20, \text{ and } .30$ (Annual Demand = \$3000)

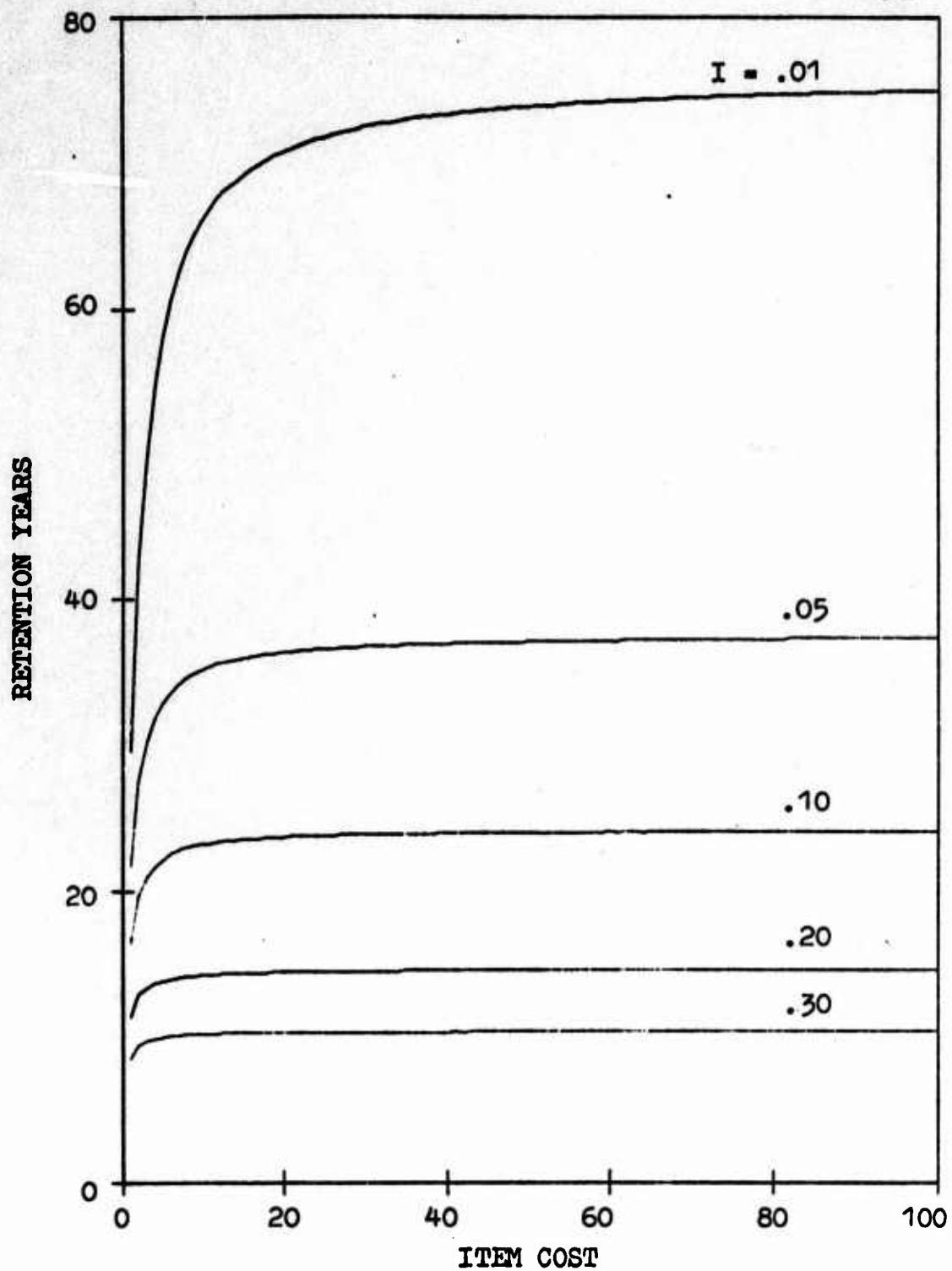


Figure 8

Retention Years Versus Item Cost For $I = .01, .05, .10, .20, \text{ and } .30$ (Annual Demand = \$15,000)

demand of \$15,000. Inspection of the figures indicated that the percent change in retention years Y for a given percent change in the opportunity cost factor J varied throughout the range of item cost C . Accordingly, three values of C were chosen to illustrate the changes in Y for a change in J . Table 5 depicts the change in Y as J increased from .1 to .23 for $C = \$1, \10 , and $\$100$. These results indicated that the model was relatively insensitive to changes in J except when C was less than $\$10$.

Net Salvage Value Factor ($K - L$)

The effect of changes in net salvage value factor ($K - L$) on the model was examined for the following parameter values:

$$I = .01$$

$$J = .23$$

$$(K - L) = .05, .10, .20, .30, .40, .50$$

$$R_1 = R_2 = 0$$

$$\$1 \leq C \leq \$100$$

$$\text{Annual Dollar Demand} = \$3000 \text{ and } \$15,000$$

Figure 11 shows the results of the analysis for a \$3000 annual demand. Results for an annual demand of \$15,000 are indicated in Figure 12. The percent change in retention years Y for a given change in the net salvage value factor ($K - L$) appeared to remain relatively constant throughout the range of item cost C . For $C = \$100$, Y changed from 76 years to 45 years, a 41 percent

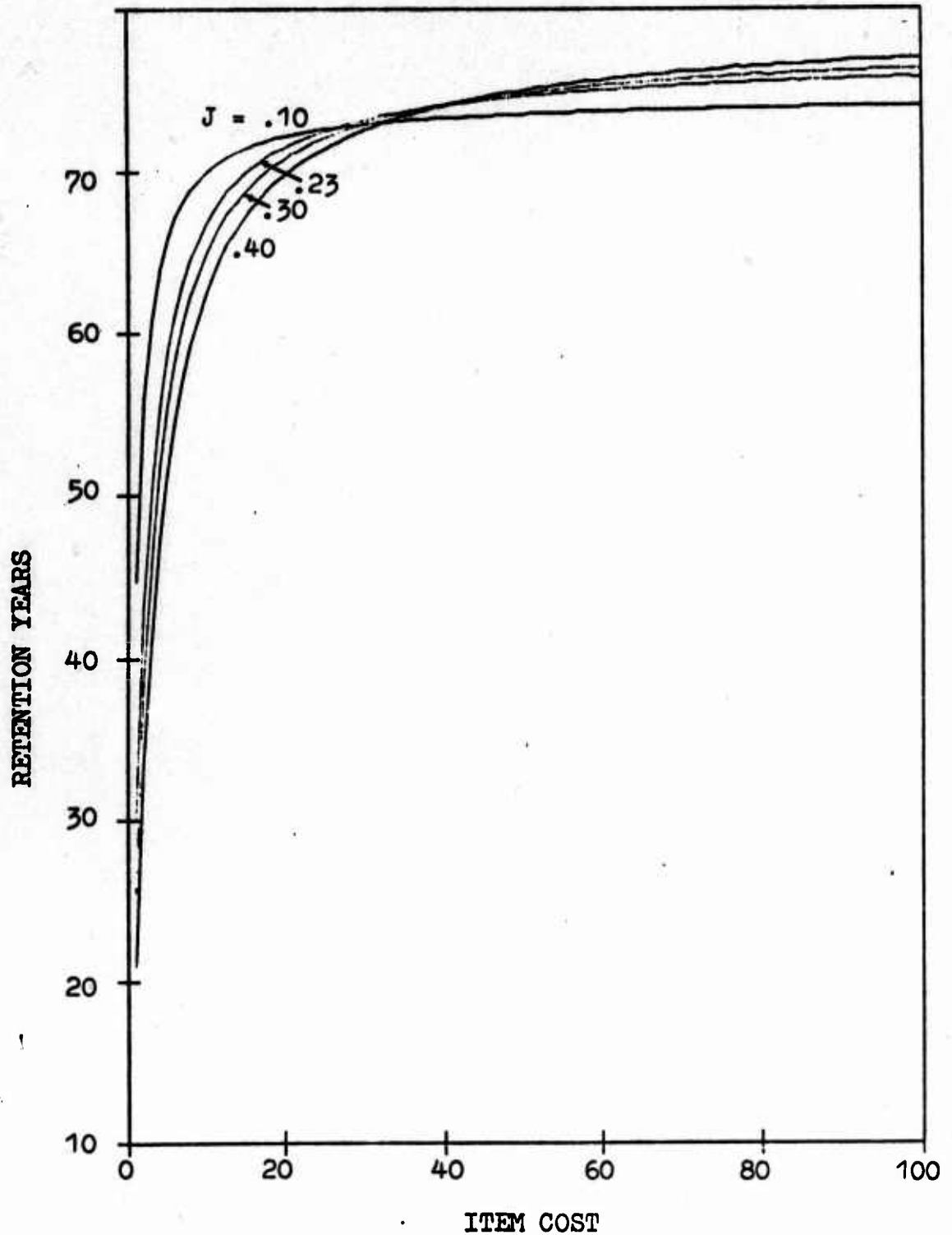


Figure 9

Retention Years Versus Item Cost For $J = .10$, $.23$,
 $.30$, and $.40$ (Annual Demand = \$3000)

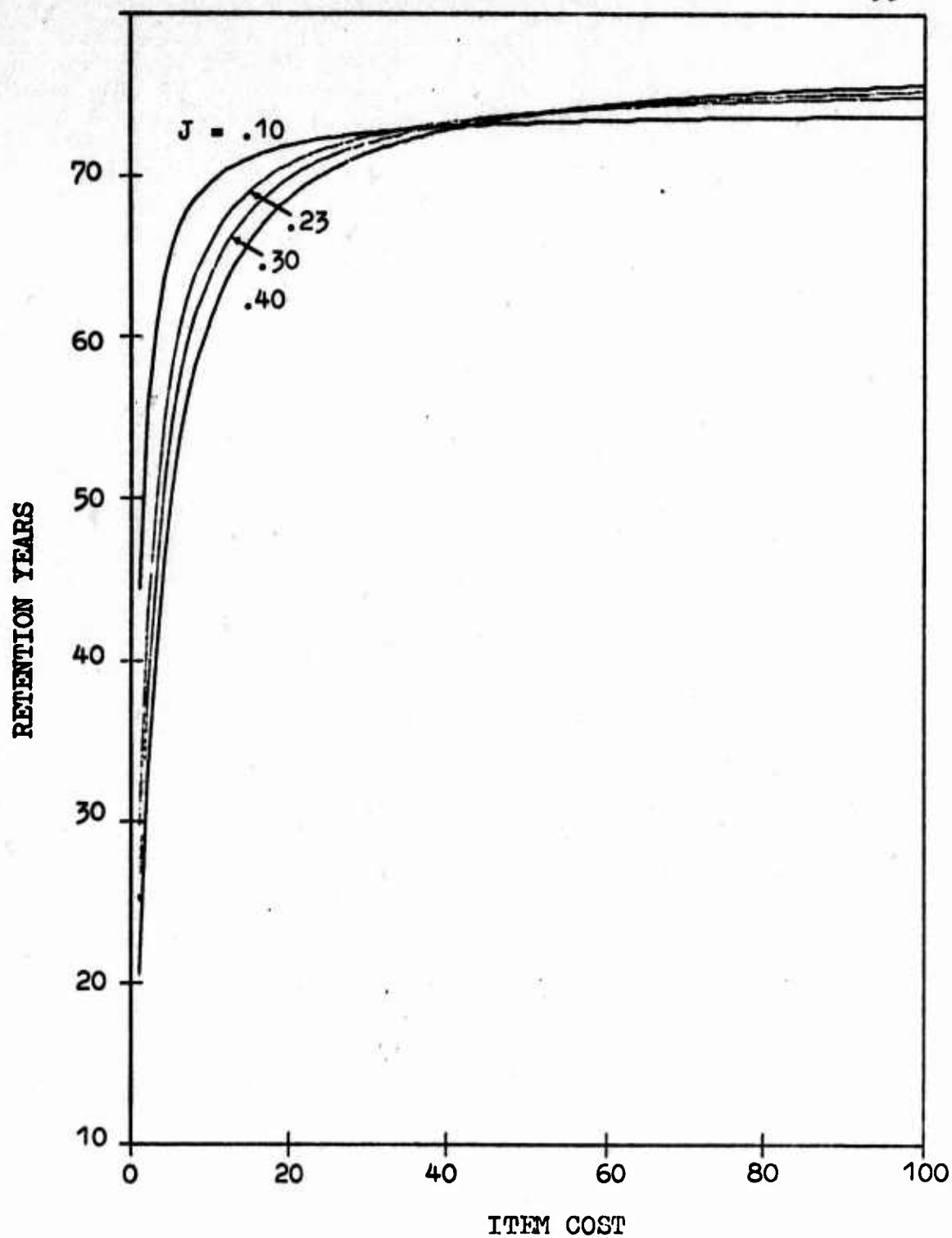


Figure 10

Retention Years Versus Item Cost For $J = .10, .23, .30$, and $.40$ (Annual Demand = \$15,000)

Table 5

Percent Change in Y For a 130 Percent Change
In J For C = \$1, \$10, and \$100

C(\$)	J		Percent Change In Y	Percent Change In J
	.1	.23		
1	45	30	33	130
10	70	67	4	130
100	74	76	3	130

change, as $(K - L)$ increased 100 percent, from .10 to .20 (see Figure 11). These results indicated that the model was relatively sensitive to changes in $(K - L)$.

Inflation Factors (X_1 and X_2)

As stated in Chapter IV, the effect of inflation can be included in the model by modifying the salvage value factor K such that

$$K' = K \frac{X_2}{X_1}$$

where $X_1 = (1 + R_1)$ --inflation factor associated with costs.

$X_2 = (1 + R_2)$ --inflation factor associated with salvage value.

R_1 , the inflation rate associated with costs, and R_2 , the inflation rate associated with salvage value, varied from zero to 15 percent as indicated in Table 1. Therefore, the ratio

$$W = \frac{X_2}{X_1}$$

varied from .87 to 1.15, and K' , the adjusted salvage value factor, varied from .87 K to 1.15 K . Figures 11 and 12 were used to analyze the sensitivity of the model to changes in X_1 and X_2 as was done in the analysis of the net salvage value factor $(K - L)$.

Table 6 shows that the sensitivity of the model increased as $(K - L)$ increased for a 15 percent increase in the inflation ratio W .

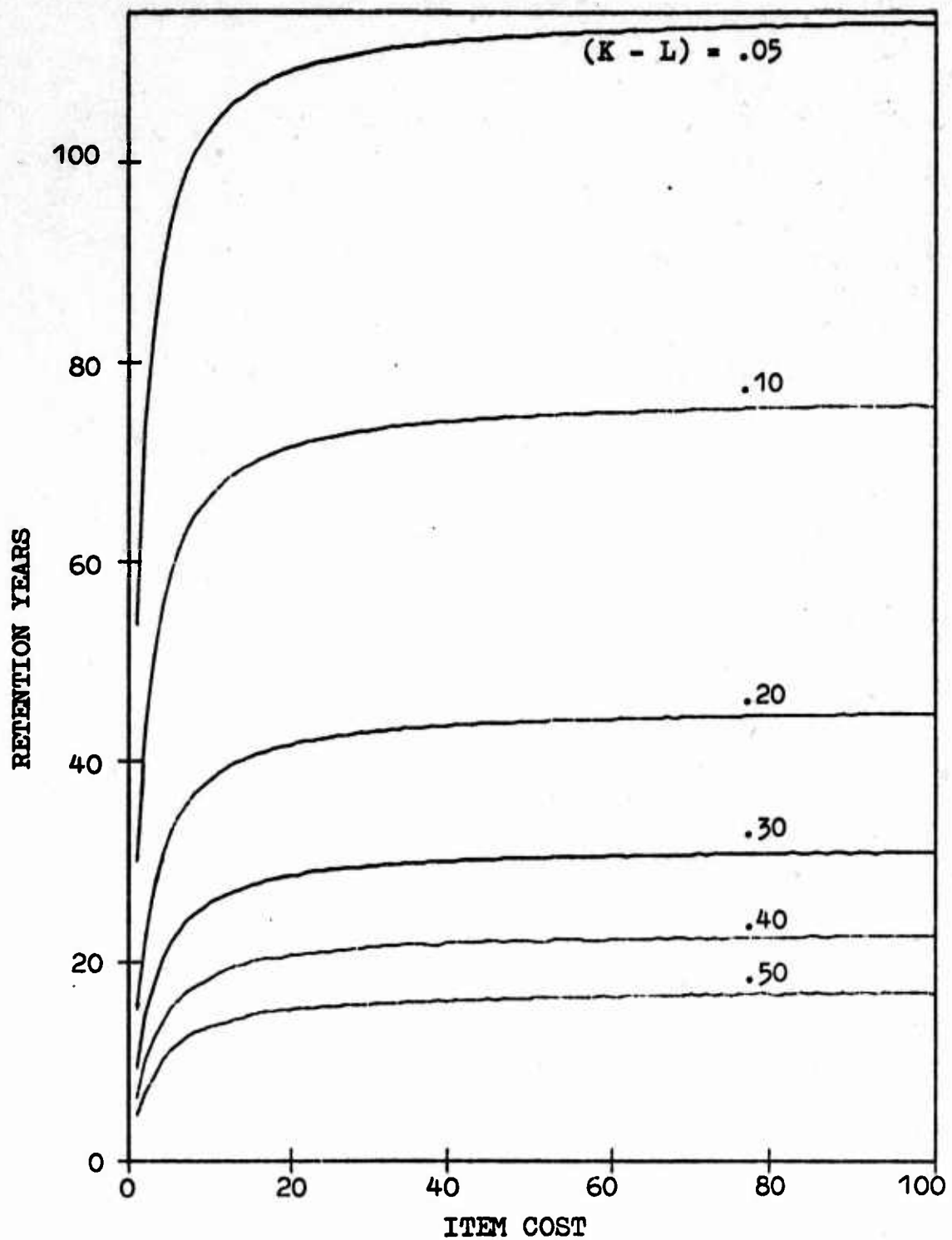


Figure 11

Retention Years Versus Item Cost For $(K - L) = .05, .10, .20, .30, .40, \text{ and } .50$ (Annual Demand = \$3000)

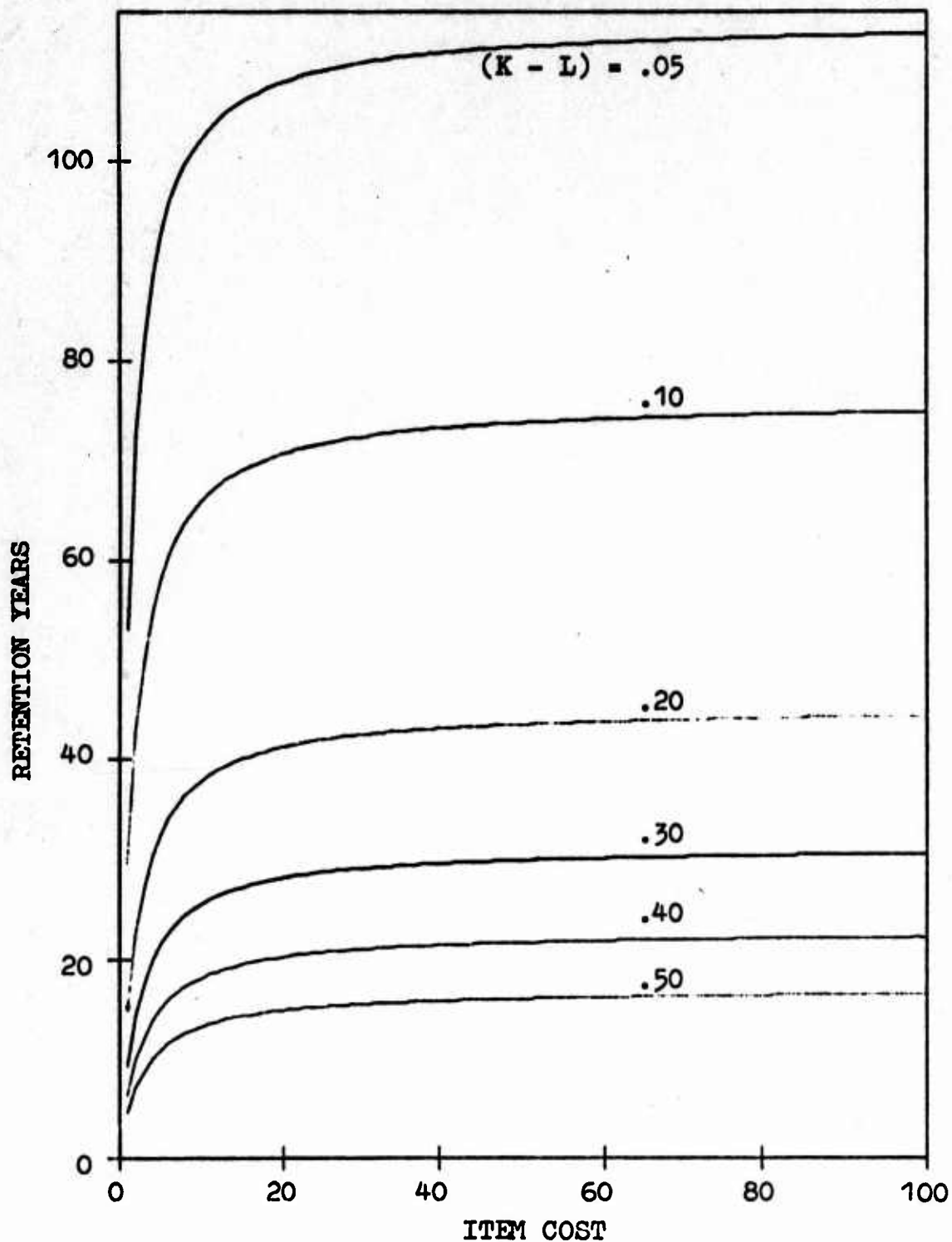


Figure 12

Retention Years Versus Item Cost For $(K - L) = .05, .10, .20, .30, .40$, and $.50$ (Annual Demand = \$15,000)

Table 6

Percent Changes In Y For a 15 Percent Change In W and
 $(K - L) = .05, .20, \text{ and } .40$ (Annual Demand = \$3000)

$(K - L)$	Y	$(K' - L)$	Y'	Change In Y	Percent Change In Y
.05	110	.0575	100	10	10
.20	43	.23	38	5	12
.40	21	.46	18	3	14

Summary of Model Sensitivity

The results of the sensitivity analysis are summarized in Table 7. The table shows that the model was relatively sensitive to the storage cost factor I and the net salvage value factor $(K - L)$ throughout the full ranges examined for all parameters. The model was relatively insensitive to order cost A and annual demand rate λ . The model was relatively sensitive to changes in item cost C for values of C less than \$10, but was relatively insensitive to changes in C when C was greater than \$10. The model was relatively insensitive to the opportunity cost factor J except for values of C less than \$10. The sensitivity of the model to inflation increased as the net salvage value factor $(K - L)$ increased. For $(K - L)$ less than or equal to .10, the model was relatively insensitive to inflation.

A SIMPLIFIED MODEL

In Chapter III, it was stated that a solution of the discounted model required an iterative procedure using a computer program. An examination of the plots of the model outputs as shown in Figures 3, 4, 5, and 6 suggested the possibility of finding a simpler algebraic function which would generate those same plots and allow solution of the model without an iterative program. When values of retention years and associated item costs were input into the CREATE System's CURFIT program (14:ST-61), the

Table 7
Sensitivity of Economic Retention Level Model

Parameter	Percent Change In Parameter	Percent Change In Retention Years
A	100	3
C		
\$1 to \$2	100	43
\$5 to \$10	100	14
\$50 to \$100	100	1
λ	200	4
I	100	40
J		
C = \$1	130	33
C = \$10	130	4
C = \$100	130	3
(K - L)	100	41
R_1, R_2		
(K - L) = .05	15	10
= .20	15	12
= .40	15	14

curves were found to fit a hyperbolic function of the form

$$(9) \quad Y = \frac{C}{(A + BC)}$$

where Y = retention years,

C = item cost,

A and B are constants generated by the CURFIT program.

Table 8 shows values of A and B from the CURFIT program for given values of annual demand in dollars.

The constants A and B were computed by the least squares method using a linear transformation of the hyperbolic equation. The CURFIT program also calculated a coefficient of determination for each set of constants, A and B , as a measure of the agreement between the actual values of retention years and the retention years calculated by Equation (9). All coefficients of determination for the sets shown in Table 8 exceeded 0.9998. Recalling that a coefficient of determination of 1.0 indicates a perfect fit (26:432) the use of Equation (9) as a substitute for Equation (8) is acceptable.

Use of Equation (9) as an expression of the discounted model allowed an algebraic analysis of the model's sensitivity to changes in item cost.

$$\begin{aligned} \frac{dY}{dC} &= \frac{(A + BC) (1) - C (B)}{(A + BC)^2} \\ &= \frac{A + BC - BC}{(A + BC)^2} \end{aligned}$$

Table 8

Values of A and B In The Hyperbolic Equation For
Item Cost Varying From \$1 to \$1000

Dollars Of Annual Demand	A	B
1000	.0194	.0125
2000	.0199	.0128
3000	.0202	.0130
4000	.0204	.0131
4900	.0205	.0131
5000	.0197	.0128
10000	.0203	.0130
15000	.0205	.0131
20000	.0207	.0132
25000	.0208	.0132
50000	.0211	.0133
75000	.0212	.0134
100000	.0212	.0134
200000	.0214	.0135

$$(10) \quad \frac{dY}{dC} = \frac{A}{(A + BC)^2}$$

Equation (10) shows that the sensitivity of the model decreases as C increases which confirms the graphical analysis conducted earlier.

The inclusion of item costs above \$1000 increased the difference between the retention years calculated by the hyperbolic equation and the retention years calculated by Equation (8). The maximum difference encountered was two percent.

Note that the independent variable in the hyperbolic equation is item cost. In effect, the constants A and B are determined by the values used for the remaining parameters. Specifically, the effects of item demand rate, order cost, net salvage value, storage cost, and opportunity cost on retention years Y are reflected in these constants. A different set of constants is obtained for each annual dollar demand value, as shown in Table 8. Therefore, A and B are parameters of the simplified model. The different values of A and B account for the different values of the parameters other than item cost used in the discounted model, Equation (8).

SUMMARY

In this chapter, a sensitivity analysis was performed on the discounted economic retention model. The model was found to be relatively sensitive to the

storage cost factor I and the net salvage value factor J. Based on observations made during the sensitivity analysis, a simpler model was presented to approximate the discounted model. The independent variable in the simpler model was item cost with parameters A and B used to account for the discounted model's parameters other than item cost. In the following chapter, conclusions based on information presented within this study are stated.

CHAPTER VI

CONCLUSIONS

The purpose of this chapter is to present the conclusions which were reached during the course of the research effort to find an economic retention level model. Some of the conclusions were based on information gathered during the literature review. Other conclusions were made after working with other researchers' models and after developing a new retention level model. A summary list of assumptions and limitations follows at the end of the chapter.

CONCLUSION 1

In Chapter I, the following research objective was stated:

The objective of this thesis is to develop a mathematical model, based on economic criteria, for determining an economic retention level for expendable type items managed by AFLC at the depot level.

The economic retention level model developed in Chapter IV satisfies the research objective.

The model can be adapted for use in the D062 System by replacing AFLC's current retention calculations as outlined in Chapter I with either the proposed discounted model or the hyperbolic model derived from the

discounted model. Since AFLC's five year retention calculations are accomplished by computer, the proposed model could be programmed to replace the programs currently being used. The validity of the proposed economic retention level model rests primarily upon its logical development from the generally accepted concepts and theories related to inventory management.

CONCLUSION 2

Interest and obsolescence charges associated with inventory holding costs should be assessed as a percentage of item salvage value. Storage costs should be assessed as a percentage of item cost.

In the economic retention model developed in Chapter IV, storage costs were assessed as a percentage of item cost. Obsolescence costs and interest charges were combined to form an opportunity cost factor which was assessed as a percentage of the net salvage value of an item. The conclusion to treat these three cost factors in the manner described was based on the definitions of the factors contained in DoD instructions (20;22;23) and on the economic considerations found to be relevant to a retention situation. The significance of this conclusion can be seen in the results produced by the model. When obsolescence and interest charges were applied to item cost, the retention level computed by the discounted model was approximately 13 years. Applying these two charges to

net salvage value produced retention levels ranging from 28 to 82 years, depending on item cost.

The decision to separate obsolescence and interest charges from storage costs was made after careful consideration of the meanings these terms had in relation to DoD inventory activities and of the economic considerations which were relevant to a retention situation. The step-by-step development of the model presented in Chapter IV and the sensitivity analysis described in Chapter V were based on generally accepted inventory management concepts and theories. The answers produced by the model should logically have no effect on the development of the model. Rather, the model itself, as a reflection of the underlying concepts and theories upon which it is based, should exemplify the relationships defined by DoD and generate results which produce inventory management decisions based on economic criteria.

On an economic basis, the difference between item cost and net salvage value of an inventory item represents a sunk cost (5:315). To assess obsolescence and interest charges on the basis of an item cost of which a portion is non-recoverable inflates the costs associated with retaining an item in inventory. Therefore, a conclusion of this study is that interest and obsolescence costs should be assessed as a percentage of item salvage value. Charging obsolescence and interest as percentages of salvage value allows a more realistic evaluation of the funds obtainable

for return to the private sector of the economy and the funds which stand to be lost in the event of item obsolescence. Assessment of costs in this manner raises item retention levels.

CONCLUSION 3

The one percent storage cost factor used by AFLC to compute economic purchase quantities may not be adequate for use in computing economic retention levels. The sensitivity analysis in Chapter V showed that the economic retention level model was relatively sensitive to changes in the storage cost factor.

The one percent storage cost value was developed for use in the EOQ model. The larger magnitude of other inventory costs relative to storage costs shifted attention away from the importance of storage costs actually incurred by a specific item or class of items (25:6;4:95). It may be necessary to develop storage cost values specifically for use in the economic retention level model to replace the one percent value currently used by AFLC.

CONCLUSION 4

The proposed economic retention level model is designed to serve as an aid to management decision-making in the retention situation and is not designed to be used in determining purchasing policy. The model reflects the economic feasibility of maintaining a quantity of items

in inventory in anticipation of future demand in lieu of disposing those items and reprocurring them at a future date. For example, an economic retention level expressed as 75 years of demand indicates that a quantity of items equal to 75 times the current forecast annual demand rate can be economically maintained in inventory. The 75 year retention level does not indicate that such a quantity should be purchased and maintained in inventory.

ASSUMPTIONS AND LIMITATIONS

1. Item demand rate is relatively stable.
2. A deterministic model is adequate to describe the behavior of the on-hand inventory position associated with the economic retention model.
3. The assumptions applicable to the Air Force EOQ model (2:17) apply also in the retention situation.
4. The average values currently used by AFLC for storage cost factor, obsolescence cost factor, and interest cost factor are valid.
5. The quantity ordered per order is strictly determined by the simple Wilson EOQ model (2:17).
6. Inventory holding costs are assessed at the end of each year. Inventory ordering costs are assessed at the time they are incurred.
7. The proposed model is limited to use with the expendable type items managed by AFLC in the D062 System which experience a stable demand rate.
8. The model applies to those items described above which have an item cost between \$1 and \$10,000 and a demand rate between 1 and 200,000 items per year.

CHAPTER VII

RECOMMENDATIONS

In the process of researching economic retention concepts and models, it became necessary to limit the scope of this study. The previous chapters have yielded conclusions applicable to retention problems associated with expendable type items. The following statements reflect recommendations concerning this area and suggest areas where further research seems warranted.

RECOMMENDATION 1

It is recommended that the model represented by Equation (8) be implemented on a field test basis. In the special case where the same values of storage cost factor I and the opportunity cost factor J are used for a large number of items, the model represented by Equation (9) can be used to compute retention levels. The computer program in Appendix B can be run for the appropriate values of I and J, and the computed retention years and associated item costs can be entered into the CURFIT program to obtain appropriate values of A and B for Equation (9).

RECOMMENDATION 2

As noted in the previous chapter, the percentage

value chosen for annual storage cost has a marked impact upon the calculated economic retention period. For this reason, further research is recommended to determine the actual magnitude of annual storage costs for groups of similar expendable type items.

RECOMMENDATION 3

Since the economic retention model developed in this study applies only to expendable type EOQ items, further research appears necessary toward developing a model for reparable items maintained in AFLC inventories.

RECOMMENDATION 4

The proposed model has shown that economic retention based on a balance of holding costs versus procurement costs yields a retention period ranging from 28 to 82 years depending on item cost. Factors other than those included in the proposed model may limit the retention period to a smaller time span. Such factors include shelf life, technological obsolescence and the availability of warehouse space. Further research is needed to assess the affects of these factors on the retention situation and to include them in the economic retention model if necessary.

APPENDIX A

VARIABLE DEFINITIONS

APPENDIX A

Variable Definitions

Model Variable Name	Fortran Variable Name	Variable Definition
Q	E	Economic order quantity
A	CO	Order cost in dollars
C	UC	Item cost in dollars
λ	M2	Annual demand rate
T_1	EY	Time between orders in years (Q/λ)
T_2	IT ₂	Economic retention period $\frac{Q + P}{\lambda}$
I	RI ₁	Storage cost factor expressed as a percentage of item cost
J	RI ₂	Opportunity cost factor (interest cost + obsolescence cost) expressed as a percentage of item cost
K	S	Salvage value factor expressed as a percentage of item cost

Model Variable Name	Fortran Variable Name	Variable Definition
L	DC	Disposal cost factor expressed as a percentage of item cost
K - L	S - DC	Net salvage value factor expressed as a percentage of item cost
m	LL	Integer portion of $\frac{T_2}{T_1}$
h		Largest integer smaller than T_2
Z		Variable representing the maximum on-hand inventory for a given demand rate and EOQ
P		Z - Q
S		Salvage value in dollars
D		Disposal cost in dollars
R ₁		Cost inflation rate expressed as a percentage of item cost
R ₂		Salvage value inflation rate expressed as a percentage of item cost

Model Variable Name	Fortran Variable Name	Variable Definition
X ₁		Cost inflation factor expressed as a percentage of item cost
X ₂		Salvage value inflation factor expressed as a percentage of item cost
K'		Salvage value factor adjusted for inflation expressed as a percentage of item cost

APPENDIX B

**COMPUTER PROGRAM FOR ECONOMIC
RETENTION LEVEL COMPUTATIONS**

APPENDIX B

COMPUTER PROGRAM FOR ECONOMIC RETENTION LEVEL COMPUTATIONS

The information contained in this appendix explains the computer program used to determine the economic retention period based on Equation (8) in Chapter IV. The program listing at the end of the appendix is the program which was used for computation of retention periods for items costing from \$1000 to \$10,000 each and having annual dollar demands of \$5000, \$10,000, \$15,000, \$20,000, and \$25,000. To investigate retention periods within other item cost ranges, line number 270 was modified to include the required item costs. Additionally, line number 310 was modified to reflect the lower order cost of \$149 if required.

Logic Used to Generate Economic Retention Years for Selected Annual Dollar Demands and Item Costs

1. Define values for storage cost, opportunity cost, salvage value and disposal cost.
2. Establish output heading titles for holding cost, order cost, holding cost versus order cost, item cost and demand.
3. Define the annual dollar demand values to which combinations of item cost multiplied by annual demand will

be set equal.

4. Beginning with an item cost of \$1000 compute the annual demand rate necessary to equal the annual dollar demand being examined. Using an order cost of \$444, the annual demand quantity, the \$1000 item cost, and the assigned values reflecting holding cost, compute an EOQ and convert it to years.

5. Using Equation (8) in Chapter IV, compute the discounted order costs and holding costs over this period. If the difference between the two discounted costs is less than zero, increase the retention period by one year and recompute the two discounted costs to include this time extension. Repeat this step until the cost difference is equal to or greater than zero.

6. When the cost difference in Step 5 is greater than zero, the point in years at which retention is economical has been exceeded. By interpolation, find the zero difference point and print the associated number of years with the information indicated by the output titles mentioned in Step 2.

7. Return to Step 4 and increase the item cost to be considered by \$250. Perform Steps 4 through 6 in \$250 increments until an economic retention period has been obtained for an item costing \$10,000.

8. Return to Step 3 and select the next annual dollar demand value to which combinations of item cost multiplied by annual demand will be set equal. Proceed

with Steps 4, 5, 6 and 7 for this new annual dollar demand value.

9. When all dollar demand values defined in the program have been investigated through use of the steps noted above, terminate the program.

FORTTRAN Variable Definitions

RI1	Storage cost expressed as a percentage of item cost
RI2	Opportunity cost expressed as a percentage of item cost
S	Salvage value expressed as a percentage of item cost
DC	Disposal cost expressed as a percentage of item cost
NX	Control variable which identifies the annual dollar demand value to be used in determining the demand rate
M9	The annual dollar demand
M1	Item cost in dollars
M2	The annual demand determined by dividing annual dollar demand by item cost
CO	Order cost
UC	Item cost in dollars
E	EOQ quantity of items
EY	EOQ quantity in years of annual demand
NN	Index counter for tracing the number of years of retention being considered in the iteration
MM	The maximum number of years of retention considered in the program. The value of MM was set arbitrarily high at 300 years
IEY	Integer value of the EOQ years stored in EY

IT2	Integer value of the retention period considered
THC	Total discounted holding costs
TOC	Total discounted order costs
SOC	Storage variable to hold the discounted order cost sums during the iterative process
HC1	Cost in dollars for holding items
HC2	Opportunity cost in dollars
HC	Discounted sum of HC1 and HC2
LL	The total number of EOQ cycles contained in the retention period considered
JJ	Index variable indicating the EOQ cycle within the retention period for which order costs are discounted during the iteration
XY	The number of years over which order costs are discounted up to JJ
OC	The discounted cost per order
DIFF	The difference between total discounted order costs and total discounted holding costs in the current iteration
IT2L	These variables contain the respective values of IT2, THC, TOC, DIFF, M1 and M2 from the previous iteration
THCL	
TOCL	
DIFFL	
M1L	
M2L	
PVF (KZ)	Function subprogram which computes present value factor applicable to holding costs
PVF (XY)	Function subprogram which computes present value factor applicable to order costs

COMPUTER PROGRAM FOR ECONOMIC RETENTION CALCULATION

```

100 RI1=.01;RI2=.23;S=.1;DC=.0
110 PRINT 120
120 120 FORMAT(//)
130 PRINT 300
140 300 FORMAT("RET YRS",2X,"HOLDING COST",7X,"ORDER COST"
150,5X,"HC - OC",7X,"UC",7X,"DMD")
160 DO 400 NX=1,5
170 GO TO(1,2,3,4,5),NX
180 1 M9=25000
190 GO TO 88
200 2 M9=20000
210 GO TO 88
220 3 M9=15000
230 GO TO 88
240 4 M9=10000
250 GO TO 88
260 5 M9=5000
270 88 DO 311 M1=1000,10000,250
280 IF(M1.GT.M9)GO TO 400
290 M2=M9/M1
300 D=M2;UC=M1
310 CO=444.
320 E=SQRT(2.*CO*D/((RI1+RI2)*UC))
330 EY=E/D
340 NN=2;MM=300
350 IEY=EY
360 IF(IEY.GT.NN)NN=IEY
370 DO 18 IT2=NN,MM
380 THC=0.;TOC=0.;SOC=0.
390 DO 15 K=1,IT2
400 KZ=K
410 HC1=(RI1*UC*(D*IT2-E-D*(K-1)))/2.
420 HC2=(D*IT2-E)*RI2*(S-DC)
430 HC=(HC1+HC2)*PVF(KZ)
440 THC=THC+HC
450 15 CONTINUE
460 LL=INT(IT2/EY)
470 DO 70 JJ=1,LL
480 XY=JJ*EY
490 OC=(UC*E+CO)*PVG(XY)
500 SOC=SOC+OC
510 70 CONTINUE
520 TOC=SOC-(D*IT2-E)*UC*(S-DC)
530 DIFF=THC-TOC
540 IF(DIFF.GT.0.)GO TO 81
550 IT2L=IT2;THCL=THC;TOCL=TOC;DIFFL=DIFF

```

```
560 M1L=M1;M2L=M2
570 18 CONTINUE
580 81 PRINT 150,IT2L,THCL,TOCL,DIFFL,M1L,M2L
590 PRINT 150,IT2,THC,TOC,DIFF,M1,M2
600 150 FORMAT(I3,9X,F10.2,6X,F10.2,4X,F10.2,4X,I6,4X,I6)
610 TRPL=ABS(DIFFL)/(ABS(DIFFL)+DIFF)
620 TRYR=IT2L+TRPL
630 PRINT 155,TRYR
640 155 FORMAT("INTERPOLATION -",F6.2)
650 PRINT 120
660 311 CONTINUE
670 400 CONTINUE
680 STOP
690 END
700 FUNCTION PVF(K)
710 PVF=1./(1.1**K)
720 RETURN
730 END
740 FUNCTION PVG(ZY)
750 PVG=1./(1.1)**ZY
760 RETURN
770 END
```

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BIOGRAPHICAL SKETCHES

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Captain Harris was born [REDACTED] where he completed his secondary education. At the Georgia Institute of Technology he was selected for membership in Alpha Pi Mu Industrial Engineering Honorary Society and graduated in March 1968 with a Bachelors Degree in Industrial Engineering. Commissioned upon graduation, he immediately entered Undergraduate Pilot Training. Subsequent assignments in the U. S. and Vietnam were as aircraft commander, instructor pilot and flight examiner. He entered AFIT, School of Systems and Logistics, in August 1974 and was selected as a member of Sigma Iota Epsilon and Alpha Iota Delta honorary management fraternities. He is married and has one son.

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